MAT598 - Additional Problem Set 01

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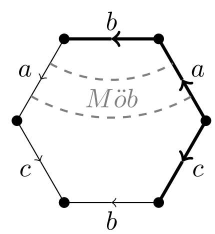
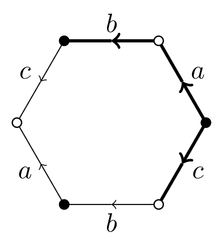
Solutions will be posted as they are submitted to me.

1.

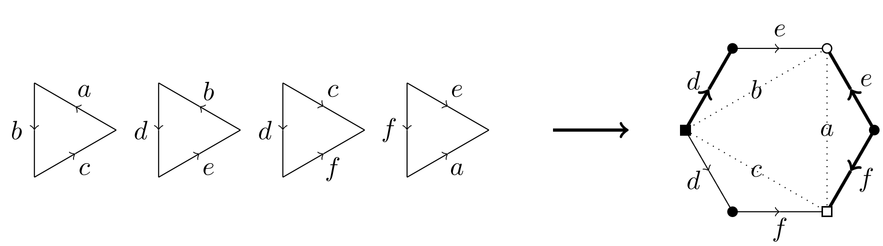
a. Let S be the surface given by the presentation. Since the a edges form a twisted pair, the surface contains a Möbius strip, and thus is nonorientable. By chasing vertices around the edges, we have that $V = 1$. Since edges are identified in pairs, we have that $E = 3$. Lastly, there is just 1 face, so $\chi(S) = V - E + F = -1$. By the classification of surfaces, we have that $S \cong 3\mathbb{P}^2 = \mathbb{P}^2\#\mathbb{P}^2\#\mathbb{P}^2$.



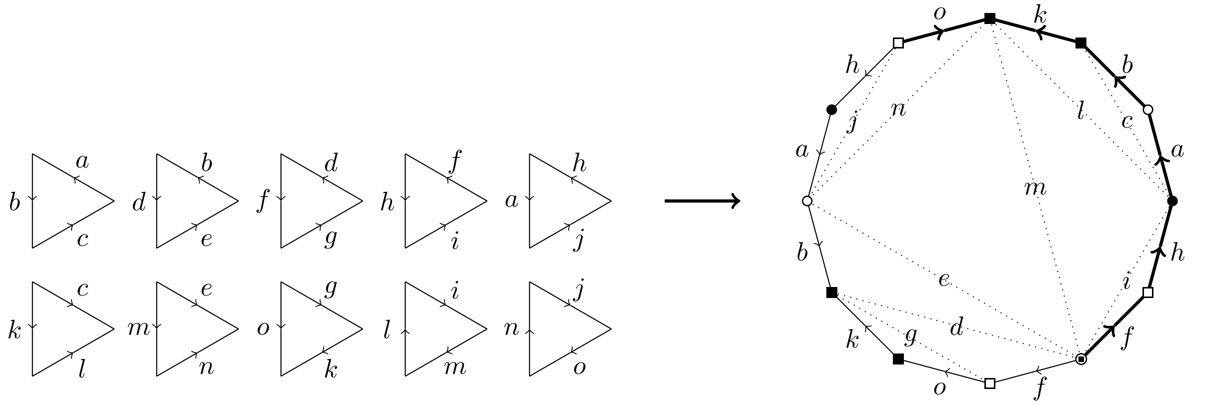
b. Let S be the surface given by the presentation. There are no twisted pairs of edges, so the surface does not contain a Möbius strip, and thus is orientable. By chasing vertices around the edges, we have that $V = 2$. Since edges are identified in pairs, we have that $E = 3$. Lastly, there is just 1 face, so $\chi(S) = V - E + F = 0$. By the classification of surfaces, we have that $S \cong \mathbb{T}^2$.



c. Although we can determine the Euler characteristic from the information given, orientability is not obvious. So, we rewrite this presentation with a single relation (this is pictorially demonstrated below) as $\langle d, e, f \,|\, ee^{-1}d^{-1}dff^{-1}\rangle$. From here it is clear that our presentation does not contain any twisted pairs of edges, so it is does not contain a Möbius strip, and thus is orientable. By chasing vertices around the edges, we have that $V = 4$. Since edges are identified in pairs, we have that $E = 3$. Lastly, there is just 1 face, so $\chi(S) = V - E + F = 2$. By the classification of surfaces, we have that $S \cong S^2$.



d. Although we can determine the Euler characteristic from the information given, orientability is not obvious. So, we rewrite this presentation with a single relation (this is pictorially demonstrated below) as $\langle a, b, f, h, k, o \,|\, abko^{-1}habk^{-1}o^{-1}f^{-1}fh\rangle$. From here it is clear that, since the a edges form a twisted pair, the surface contains a Möbius strip, and thus is nonorientable. By chasing vertices around the edges, we have that $V = 5$. Since edges are identified in pairs, we have that $E = 6$. Lastly, there is just 1 face, so $\chi(S) = V - E + F = 0$. By the classification of surfaces, we have that $S \cong \mathbb{P}^2 \# \mathbb{P}^2$.



2. *INCOMPLETE*

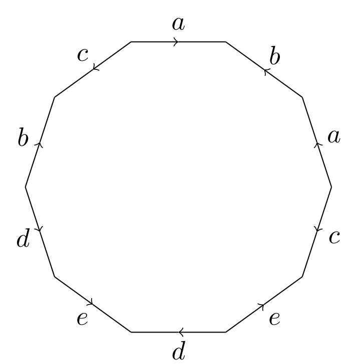
3.

a. *INCOMPLETE*

b. *INCOMPLETE*

4. *INCOMPLETE*

5.



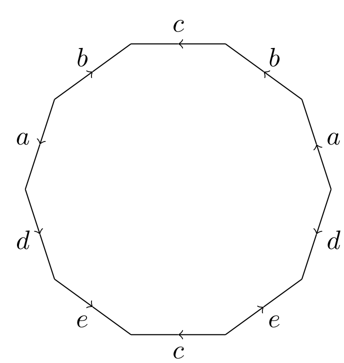
a. *INCOMPLETE*

b. *INCOMPLETE*

c. *INCOMPLETE*

d. *INCOMPLETE*

6.



- **a.** *INCOMPLETE*
- **b.** *INCOMPLETE*
- **c.** *INCOMPLETE*
- **d.** *INCOMPLETE*

7. *INCOMPLETE*