MAT502 - Additional Problem Set 07

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1. State the Tensor Characterization Lemma (as it relates to smooth vector fields).

2. Suppose (M, g) is a Riemannian manifold. A smooth curve $\gamma : (a, b) \to M$ is said to be ***unit-speed*** if $|\gamma'(t)|_g \equiv 1$. Prove that every smooth curve with nowhere-nonvanishing velocity has a unit-speed reparametrization.

3. *[Spring 2015, Problem 8]* Let (M, g) be a smooth n-dimensional Riemannian manifold.

- **a.** Let N be a smooth manifold and $F : N \to M$ be a smooth map. Prove that $h = F^*g$ is a smooth metric on N if and only if F is an immersion.
- **b.** Part (a), together with a certain big theorem, implies that every smooth manifold admits a Riemannian metric. State(some version of) this big theorem.
- **c.** Use the existence of smooth partitions of unity to give another proof that every manifold admits a Riemannian metric.
- **d.** Prove that if $n = 1$, (M, g) is locally isometric to $(\mathbb{R}, g_{\text{euc}})$.