# MAT502 - Additional Problem Set 06

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**1.** True or false? Every smooth covector field on $\mathbb{R}$ is exact.

**2.** For any smooth manifold $M$, show that $T^*M$ is a trivial vector bundle if and only if $TM$ is trivial.

**3.** Prove that if $\varphi : [c,d] \to [a,b]$ is a decreasing diffeomorphism, then

$$\int_{[c,d]} \varphi^* \omega = -\int_{[a,b]} \omega.$$

**4.** *[Spring 2015, Problem 5]* Let $M$ be a smooth manifold and $f \in C^\infty(M)$.

- **a.** Give the definition of the differential $df$ of $f$.
- **b.** Suppose that $M$ is connected. Prove that $M$ is constant on $M$ if and only if $df = 0$.
- **c.** State and prove the "fundamental theorem" for line integrals. [This is the theorem which expresses the value of $\int_\gamma df$ for a smooth function $f$ and a piecewise smooth path $\gamma$.]

**5.** For a smooth real-valued function $f : M \to \mathbb{R}$, show that $p \in M$ is a critical point of $f$ if and only if $df_p = 0$.

**6.** Let $M$ be a smooth manifold without boundary. Prove that a smooth covector field $\omega \in \mathfrak{X}^*(M)$ is conservative if and only if, given two piecewise smooth curves $\gamma, \tilde{\gamma} : [a,b] \to M$ with $\gamma(a) = \tilde{\gamma}(a)$ and $\gamma(b) = \tilde{\gamma}(b)$, we have $\int_\gamma \omega = \int_{\tilde{\gamma}} \omega$.