MAT598 - Additional Problem Set 02

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Solutions will be posted as they are submitted to me.

1. *INCOMPLETE*

2.

  a. *INCOMPLETE*

  b. *INCOMPLETE*

  c. *INCOMPLETE*

3. *INCOMPLETE*

4. *INCOMPLETE*

5. *INCOMPLETE*

6. *INCOMPLETE* Let $\varphi : \pi_1(X \times Y) \to \pi_1(X) \times \pi_1(Y)$ be given by $\varphi : [f] \mapsto (p_{1*}[f], p_{2*}[f])$, where $p_1 : X \times Y \to X$ and $p_2 : X \times Y \to Y$ are canonical projections. We note that the induced homomorphisms π_{1*} and π_{2*} are well-defined, and so φ is as well. It follows as well that φ is a homomorphism.

Let g_1 be a loop in X and g_2 a loop in Y. Define a loop f in $X \times Y$ by $f(t) = (g(t), h(t))$. We then have that $g_1(t) = p_1(f(t))$ and $g_2(t) = p_2(f(t))$. As such, we now have that $\varphi([f]) = ([g_1], [g_2])$ and thus φ is surjective.

Finally, let c_1, c_2 be constant loops in X and Y, respectively, and suppose, for $[f] \in \pi_1(X \times Y)$, we have that $\varphi([f]) = ([c_1], [c_2])$. Then we must have $p_{1*}[f] = [p_1(f)] = [c_1]$ and $p_{2*}[f] = [p_2(f)] = [c_2]$, so $[f] = [(c_1, c_2)]$ and hence f is homotopic to the constant loop in $\pi_1(X \times Y)$. Thus, $\ker \varphi$ is trivial. By the first isomorphism theorem for groups, we have

$$\pi_1(X \times Y) \equiv \pi_1(X \times Y)/\ker \varphi \equiv \operatorname{Im} \varphi = \pi_1(X) \times \pi_1(Y).$$