MAT598 - Additional Problem Set 03

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1. *[May 2015]* Prove that the Möbius band does not retract onto its boundary circle.

2. *[August 2015]* Given $x_0 \in S^1$, consider the subspaces $C_1 = \{x_0\} \times S^1$ and $C_2 = S^1 \times \{x_0\}$ of the torus $T^2 = S^1 \times S^1$, and a point $p \in T^2 - (C_1 \cup C_2)$.

- **a.** Does T^2 retract (resp. deformation retract) onto C_1 or C_2?
- **b.** Does T^2 retract (resp. deformation retract) onto $C_1 \cup C_2$?
- **c.** Does $T^2 - \{p\}$ retract (resp. deformation retract) onto $C_1 \cup C_2$?

3. For spaces $X \subseteq Y \subseteq Z$, suppose that Y is a retract of Z and Z deformation retracts onto X. Show that X is a deformation retract of Y.

4. Suppose that a space X deformation retracts onto a subspace X_0 and we attach X to a space Y along a subspace $A \subseteq X_0$ via the map $f : A \to Y$ to form a space $Z = Y \sqcup_f X$. Show that Z deformation retracts onto $Z_0 = Y \sqcup_f X_0$.

5. Given a space X and a path-connected subspace A containing the basepoint x_0, show that the map $\pi_1(A, x_0) \to \pi_1(X, x_0)$ induced by the inclusion $A \hookrightarrow X$ is surjective iff every path in X with endpoints in A is homotopic to a path in A.

6. Show that the isomorphism $\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y)$ is given by $[f] \mapsto (p_{1*}([f]), p_{2*}([f]))$, where p_1 and p_2 are projections of $X \times Y$ onto its two factors.