Name:________________________

§12.4 Partial Derivatives

1. Find all first-order partial derivatives of $f(x, y, z) = \dfrac{2}{\sqrt{x^2 + y^2 + z^2}}$

2. Find the partial derivatives f_{xx}, f_{yy}, and f_{xy} of the function $f(x, y, z) = x^2y - 4x + 3\sin(y)$.

3. Find all points at which $f_x = f_y = 0$ for the function $f(x, y) = e^{-x^2-y^2}$ and interpret the significance of the points graphically.

4. For any positive integer n and constant c, show that the functions $f_n(x,t) = \sin(n\pi x)\cos(cn\pi t)$ satisfy the wave equation: $c^2 f_{xx} = f_{tt}$.

5. The Ideal Gas Law relating pressure (P), temperature (T), and volume (V) is $P = \dfrac{cT}{V}$ for some constant c. Show that $TP_{_T}V_{_T} = c$.

6. Suppose that L hours of labor and K dollars of investment by a company result in a productivity of $P = L^{0.75}K^{0.25}$. Compute the marginal producitivity of labor $P_{_L}$, and the marginal productivity of capital $P_{_K}$.

§12.5 The Chain Rule

7. Use the chain rule to find f_u and f_v for $f(x,y) = xy^3 - 4x^2$, where $x(u,v) = e^{u^2}$ and $y(u,v) = \sin(u)\sqrt{v^2+1}$.

8. State the chain rule for the general composite function $g(u,v) = f(x(u,v), y(u,v), z(u,v))$.

9. Suppose the production of a firm is modeled by $P(k,\ell) = 16k^{1/3}\ell^{2/3}$, where K measures capital (in millions of dollars) and ℓ measures the labor force (in thousands of workers). Suppose that $\ell = 2$ and $k = 5$, the labor force is increasing at the rate of 40 workers per year, and capital is decreasing at the rate of $\$100,000$ per year. Determine the rate of change of production.

10. Use the chain rule twice to find $g''(t)$, where $g(t) = f(x(t), y(t), z(t))$.

11. Use implicit differentiation to find z_x and z_y, where $3yz^2 - e^{4x}\cos(4z) - 3y^2 = 4$.

12. Write out the third-order Taylor polynomial for $f(x, y) = \sin(xy)$ about $(0, 0)$.

§12.6 Directional Derivatives and the Gradient

13. Find the gradient of $f(x, y) = 3\sin(3xy) + y^2$ at $(\pi, 1)$.

14. Compute the directional derivative of $f(x, y) = e^{4x^2-y}$ at the point $(1, 4)$ in the direction of the vector $\langle -2, -1\rangle$.

15. Find the directions of maximum and minimum change of f at the given point, and the values of the maximum and minimum rates of change: $f(x, y) = x^2 - y^2$ at $(-2, -1)$.

16. Find equations of the tangent plane and normal line to the surface $f(x,y) = \sqrt{x^2+y^2}$ at $(3,-4,5)$.

17. Find all points at which the tangent plane to the surface $z = \sin(x)\cos(y)$ is parallel to the xy-plane.

18. If the temperature at the point (x,y,z) is given by $T(x,y,z) = 80 + 5e^{-z}(x^{-2}+y^{-1})$, find the direction from the point $(1,4,8)$ in which the temperature decreases most rapidly.