Recitation 13: Taylor Series and Parametric Equations

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Example (Rec Ntbk §9.3, #3).

a. Find the first four nonzero terms of the Taylor series centered at 2 for the function $f(x) = \frac{1}{x}$.

b. Write the power series using summation notation.

$$p_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$$

In summation notation, this is

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{k+1}}(x-2)^k$$

Example (Rec Ntbk §9.4, #1)**.** Evaluate the limit using the Taylor series: $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$.

Use the substitution $x = \frac{1}{t}$ and note that $x \to \infty$ as $t \to 0^+$. Also, let's pretend we know nothing about the sinc function or L'Hopital's. (Aside: $\text{sinc}(x) = \frac{\sin(x)}{x}$, and we call it the "sinc" or "cardinal sine" function.)

$$\begin{aligned}
\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) &= \lim_{t\to 0^+} \frac{\sin(t)}{t} \\
&= \lim_{t\to 0^+} \frac{\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} + \cdots\right)}{t} \\
&= \lim_{t\to 0^+} \left(1 - \frac{t^2}{3!} + \frac{t^4}{5!} + \cdots\right) \\
&= 1.
\end{aligned}$$

Long ago, we noticed that functions $y = f(x)$ were not perfect for modeling everything we might want to model - in particular, it wasn't useful for graphs that aren't properly functions. However, we could describe the x- and y-directions separately as a function of time, we could expand our modeling ability greatly. The bonus, also, is that it allows us to put an orientataion on our graph (that is, label the figure so that it represents the motion as time increases.

The following examples show (working backwards), how common figures can be presented parametrically.

Example (Book §10.1, # 11 and # 12).

a. Eliminate the Parameter to obtain an equation in x and y.

b. Describe the curve and indicate the positive orientation

11. $x = \sqrt{t} + 4,\ y = 2\sqrt{t};\ 0 \le t \le 16.$

12. $x = (t+1)^2,\ y = t + 2;\ -10 \le 10 \le 10$

11. Let $\sqrt{t} = \frac{y}{3}$. Then

$$x = \sqrt{t} + 4 = \frac{y}{3} + 4$$
$$\Rightarrow y = 3x - 4.$$

This is just a line.

12. Let $y - 1 = t + 1$. Then

$$\begin{aligned} x &= (t+1)^2 \\ &= (y-1)^2 \end{aligned}$$

This is just a (horizontal) parabola.

Taking a derivative is easy in parametric equations too. Notice that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Example (Book §10.1, #47)**.**

a. Determine $\frac{dy}{dx}$ in terms of t and evaluate it at the given value of t.

b. Make a sketch of the curve showing the tangent line at the point corresponding to the given value of t.

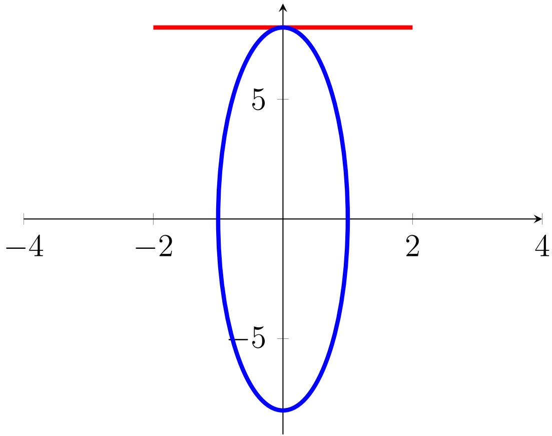
$x = \cos(t)$, $y = 8\sin(t)$, $t = \frac{\pi}{2}$.

By our previous equation

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{8\cos(t)}{-\sin(t)}$$

so

$$\left.\frac{dy}{dx}\right|_{t=\pi/2} = \frac{8\cos\left(\frac{\pi}{2}\right)}{-\sin\left(\frac{\pi}{2}\right)} = 0$$



5
−4
−2
2
4
−5

Assignment

Due (Wed) April 30 / (Fri) May 2

Recitation Notebook:
§9.3 - # 2
§9.4 - # 2
§10.2 - # 2
§10.3 - # 3
§10.4 - # 4

As always, you may work in groups, but every member must individually submit a homework assignment.