Recitation 09: Sequences and Series

Joseph Wells
Arizona State University

March 17, 2014

Example (Rec Notebk: §8.1, #5). **Limits from Graphs** Consider the following sequence: $a_n = \frac{n^2}{n^2-1}$; $n = 2, 3, 4$.

a. Find the first four terms of the sequence.

b. Based on part (a) and the figure, determine a plausible limit of the sequence.

a. $\frac{4}{3}, \frac{9}{8}, \frac{16}{15}, \frac{25}{24}$

b. It looks like it is approaching 1, and in fact, it is.

Example (Rec Notebk: §8.1, #6). **Repeating decimals**

a. Write the following repeating decimal as an infinite series: $0.\overline{027} = 0.027027027\ldots$.

b. Find the limit of the sequence of partial sums for the infinite series and express it as a fraction.

a. $0.\overline{027} = \sum_{k=1}^{\infty} 27(0.001)^k$.

b. We can rewrite the series as

$$\begin{aligned}
\sum_{k=1}^{\infty} 27(0.001)^k &= -27 + \sum_{k=0}^{\infty} 27(0.001)^k \\
&= -27 + \sum_{k=0}^{\infty} 27\left(\frac{1}{1000}\right)^k \\
&= -27 + \frac{27}{1 - \frac{1}{1000}} \\
&= \frac{27}{999} = \frac{1}{37}
\end{aligned}$$

Example (Rec Notebk: §8.2, #3)**. Geometric sequences** Determine whether the following sequences converge or diverge and describe whether they do so monotonically or by oscillation. Give the limit when the sequence converges.

a. $\{(-1.01)^n\}_{n\in\mathbb{Z}^+}$

b. $\{2^n 3^{-n}\}_{n\in\mathbb{Z}^+}$

a. $\{(-1.01)^n\}$ diverges since $|-1.01| > 1$. Since it is negative, it oscillates.

b. $\{2^n 3^{-n}\} = \{(2\cdot 3^{-1})^n\} = \{(2/3)^n\}$ converges since $|2/3| < 1$. Since it is positive, it does so monotonically. The limit is 0.

Example (Book: §8.2, #18)**.** Determine the limit of the following sequence, or state that it diverges: $\{\frac{\ln(1/n)}{n}\}_{n\in\mathbb{Z}^+}$

By L'Hopital's rule,

$$\lim_{n\to\infty}\frac{\ln(1/n)}{n} \overset{L'H}{=} \lim_{n\to\infty}\frac{1}{n} = 0.$$

Theorem (8.6)**.** *The following sequences are ordered according to the increasing growth rates as $n \to \infty$. For all positive real numbers p, q, r, s and $b > 1$,*

$$\{\ln^q n\} \ll \{n^p\} \ll \{n^p \ln^r n\} \ll \{n^{p+s}\} \ll \{b^n\} \ll \{n!\} \ll \{n^n\}.$$

Example (Rec Notebk: §8.2, #4)**. Comparing growth rates of sequences** Determine which sequence has the greater growth rate as $n \to \infty$. Be sure to justify and explain your work: $a_n = 3^n$; $b_n = n!$.

Following from Theorem 8.6, $n! \gg 3^n$ since $n! \gg b^n$ for every $b > 1$.

Assignment

Recitation Notebook:
§8.1 - #1, #2, #3, #4
§8.2 - #1, #2

As always, you may work in groups, but every member must individually submit a homework assignment.