Recitation 04: Recap Trig Subs & Partial Fractions

Joseph Wells
Arizona State University

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QUIZ TIME! It's been a week, let's see if you can remember how to do this

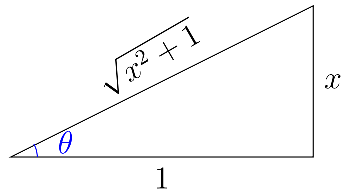
Example. Find $\displaystyle\int \frac{dx}{(x^2+1)^{3/2}}$.

Solution.

Begin by rewriting the integral to involve a square root. Then use the substitution $x = \tan(\theta)$ and $dx = \sec^2(\theta)\,d\theta$.

$$\begin{aligned}
\int \frac{dx}{(x^2+1)^{3/2}} &= \int \frac{dx}{(\sqrt{x^2+1})^3} \\
&= \int \frac{\sec^2(\theta)}{(\sqrt{\tan^2(\theta)+1})^3}\,d\theta \\
&= \int \frac{d\theta}{\sec(\theta)} \\
&= \int \cos(\theta)\,d\theta \\
&= \sin(\theta) + C
\end{aligned}$$

and our reference triangle is



Thus, in terms of x, our integral becomes

$$\int \frac{dx}{(x^2+1)^{3/2}} = \frac{x}{\sqrt{x^2+1}} + C.$$

Example. Preform partial fraction decomposition for $\dfrac{8x^3+13x}{(x^2+2)^2}$

Solution.

Since we have repeating factors and irreducible quadratics, we get

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}.$$

Multiplying by the lcd $(x^2+2)^2$ yields

$$\begin{aligned} 8x^3+13x &= (Ax+B)(x^2+2)+Cx+D \\ &= Ax^3+Bx^2+(2A+C)x+(2B+D). \end{aligned}$$

So we solve the following system of equations

$$\begin{array}{rrrrcr} Ax^3 & & & & = & 8x^3 \\ & Bx^2 & & & = & 0 \\ 2Ax & & +Cx & & = & 13x \\ & 2B & & +D & = & 0 \end{array}$$

and get that $A = 8, B = 0, C = -3, D = 0$.

Turns out, we can also use linear algebra and RREF of a matrix to solve for systems of equations (this makes it easier if we have a considerably more complex partial fraction decomposition.

Since there are four unknowns, we will have a 4x5 *augmented matrix*, with the four left columns corresponding to coefficients on A, B, C, D, respecively, and the fifth column (from the top down) containing the coefficients for the cubic, quadratic, linear, and constant terms (respectively). So, the coefficient matrix becomes

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 13 \\ 0 & 2 & 0 & 1 & 0 \end{pmatrix}$$

and when we put it into *reduced row echelon form* (RREF), we get

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

If you look, you'll see that this matrix represents the exact solutions that we got before with our usual algebraic steps.

You can preform this on a calculator as well. Here are directions for a TI-83/84:

`http://www.itc.csmd.edu/mth/ti83/matrix/rref.htm`

Example (Rec Notebk: §7.4, # 4)**.** Evaluate the following integral. $\displaystyle\int \frac{z+1}{z(z^2+4)}\,dz$

Solution.

We first preform the partial fraction decomposition

$$\begin{aligned} \frac{z+1}{z(z^2+4)} &= \frac{A}{z} + \frac{Bz+C}{z^2+4} \\ \Rightarrow \quad z+1 &= Az^2 + 4A + Bz^2 + Cz. \end{aligned}$$

Our coefficient matrix yields

$$\text{rref}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

So

$$
\begin{aligned}
\int \frac{z+1}{z(z^2+4)}\,dz &= \int \left(\frac{\frac{1}{4}}{z} + \frac{\frac{1}{4}z+1}{z^2+4} \right)\,dz \\
&= \int \frac{\frac{1}{4}}{z}\,dz + \int \frac{\frac{1}{4}z+1}{z^2+4}\,dz \\
&= \frac{1}{4}\int \frac{dz}{z} + \frac{1}{4}\int \frac{z}{z^2+4}\,dz + \int \frac{dz}{z^2+4} \\
&= \frac{1}{4}\ln|z| + \frac{1}{8}\ln|z^2+4| + \frac{1}{2}\arctan\left(\frac{z}{2}\right) + C.
\end{aligned}
$$

Assignment

Recitation Notebook:
§7.3 - #2, #4,
§7.4 - #1, #2, #5
§7.5 - #1

* For §7.4 - #2, #5 write down the augmented coefficient matrix and the reduced row echelon form (RREF) of the matrix. If you so not have access to a calculator to preform this operation, the following website can do it:

`http://www.math.purdue.edu/~dvb/matrix.html`

As always, you may work in groups, but every member must individually submit a homework assignment.