Recitation 03: Trig Substitution

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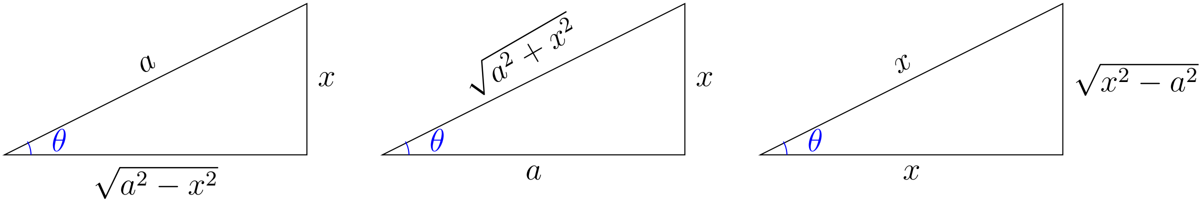
When we see a polynomial under a square root, often times it is easiest to approach with a trig substitution. Your book builds it up more intuitively, but the results for the three basic trig substitutions are as follows:

Integrand	Trig Substitution
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$.

Each of these corresponds to a reference triangle



Example (Rec Notebk: §7.3, # 1)**.** Evaluate the following integral: $\displaystyle\int \frac{dx}{\sqrt{16+4x^2}}$

Solution.

Here $a = 2$, so we use the trig substitution $x = 2\tan(\theta)$ and $dx = 2\sec^2(\theta)\,d\theta$:

$$\begin{aligned}
\int \frac{dx}{\sqrt{16+4x^2}} &= \int \frac{dx}{2\sqrt{4+x^2}} \\
&= \int \frac{2\sec^2(\theta)d\theta}{2\sqrt{4(1+\tan^2(\theta)}} \\
&= \frac{1}{2}\int \frac{\sec^2(\theta)d\theta}{\sec(\theta)} \\
&= \frac{1}{2}\int \sec(\theta)\,d\theta \\
&= \frac{1}{2}\ln|\sec(\theta)+\tan(\theta)|+C
\end{aligned}$$

Using our reference triangle, we get that $\sec(\theta) = \frac{\sqrt{4+x^2}}{2}$ and $\tan(\theta) = \frac{x}{2}$, so

$$\frac{1}{2}\ln|\sec(\theta) + \tan(\theta)| + C = \frac{1}{2}\ln\left|\frac{1}{2}(\sqrt{4+x^2} + x)\right| + C.$$

Assignment

This week's worksheet can be found here:

`http://math.joedub.net/teaching/mat271_spring2014/mat271_spring2014_homework03.pdf`

As always, you may work in groups, but every member must individually submit a homework assignment.