

# MAT271 Exam 3 Review

Spring 2014

*This is not a complete list of topics covered in class, but merely a compilation of supplemental exercises from each section. You should still review class notes and the practice exam posted on ASU's MAT271 course page:*

<http://math.asu.edu/first-year-math/mat-271-calculus-analytic-geometry-ii>

## §8.1 An Overview (of Sequences)

For each of the following sequences,

- Find the next two terms in the sequence.
- Find a recurrence relation that generates the sequence (supply the initial value of the index and the first term of the sequence).
- Find an explicit formula for the general  $n^{\text{th}}$  term of the sequence.

1.  $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$

2.  $\{1, -2, 3, -4, 5, \dots\}$

3.  $\{1, 2, 4, 8, 16, \dots\}$

4.  $\{1, 3, 9, 27, 81, \dots\}$

## §8.2 Sequences

Find the limit of the following sequences, or state that the limit does not exist. If the sequence is geometric, determine whether or not the sequence is monotone.

5.  $\left\{\frac{2e^{n+1}}{e^n}\right\}$

6.  $\left\{\left(\frac{1}{n}\right)^{1/n}\right\}$

7.  $\{(-0.003)^n\}$

8.  $\left\{\frac{n \sin^3(n)}{n+1}\right\}$

## §8.3 Infinite Series

For each of the following sums/series, determine the type of sum/series. Then evaluate the sum/series, or state that it diverges.

9.  $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \dots + \frac{1}{2916}$

10.  $\sum_{m=2}^{\infty} \frac{5}{2^m}$

11.  $\sum_{k=1}^{\infty} 3 \left(\frac{-1}{8}\right)^{3k}$

12.  $\sum_{k=1}^{\infty} [\tan^{-1}(k+1) - \tan^{-1}(k)]$

## §8.4 The Divergence and Integral Tests

Determine whether or not the following series converge or diverge, and state the test used.

$$13. \sum_{k=1}^{\infty} \sqrt{\frac{k+1}{k}}$$

$$15. \sum_{k=0}^{\infty} \frac{10}{k^2 + 9}$$

$$14. \sum_{k=1}^{\infty} \frac{1}{(3x+1)(3x+4)}$$

$$16. \sum_{k=2}^{\infty} \frac{4}{k \ln^2(k)}$$

## §8.5 The Ratio, Root, and Comparison Tests

Determine whether or not the following series converge or diverge, and state the test used.

$$17. \sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$$

$$19. \sum_{k=1}^{\infty} \sin^2\left(\frac{1}{k}\right)$$

$$18. \sum_{k=2}^{\infty} \frac{1}{k^2 \ln(k)}$$

$$20. \frac{1}{1!} + \frac{4}{2!} + \frac{9}{3!} + \frac{16}{4!} + \dots$$

## §8.6 Alternating Series

Determine whether or not the following series converge or diverge.

$$21. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^{10} + 2k^5 + 1}{k(k^{10} + 1)}$$

$$23. \sum_{k=1}^{\infty} \frac{(-1)^k k}{2k + 1}$$

$$22. \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2 + 4}}$$

$$24. \sum_{k=1}^{\infty} \frac{(-1)^k \tan^{-1}(k)}{k^3}$$

## §9.1 Approximating Functions with Polynomials

Find the  $n^{\text{th}}$ -order Taylor polynomial for the given functions centered at the given point  $a$ , where  $n = 0, 1, 2$ .

$$25. f(x) = \ln(1-x), a = 0$$

$$27. f(x) = \sin(x), a = \frac{\pi}{4}$$

$$26. f(x) = \tan(x), a = 0$$

$$28. f(x) = \sqrt{x}, a = 9$$

## §9.2 Properties of Power Series

Find the interval and radius of convergence for each of the following power series.

$$29. \sum \frac{k^2 x^{2k}}{k!}$$

$$30. \sum \frac{(x-1)^k k^k}{(k+1)^k}$$

Find the power series representation for  $g$  centered at 0 by differentiating or integrating the power series for  $f$  (perhaps more than once). Give the interval of convergence for the resulting series.

$$31. g(x) = \frac{1}{(x-4)^4} \text{ using } f(x) = \frac{1}{1-x}$$

$$32. g(x) = \ln(1-3x) \text{ using } f(x) = \frac{1}{1-3x}$$

## §9.3 Taylor Series

For the following series...

- Find the first four nonzero terms of the Taylor series centered at  $a$ .
- Write the power series using summation notation.

$$33. f(x) = (1+x^2)^{-1}, a = 0$$

$$35. f(x) = \sin(x), a = \frac{\pi}{2}$$

$$34. f(x) = \tan^{-1}(x), a = 0$$

$$36. f(x) = \ln(x), a = 3$$

## §9.4 Working with Taylor Series

Evaluate the following limits using Taylor series.

$$37. \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

$$39. \lim_{x \rightarrow 0} \frac{\sin(x) - \tan(x)}{3x^3 \cos(x)}$$

$$38. \lim_{x \rightarrow 0} \frac{3 \tan(x) - 3x - x^3}{x^5}$$

$$40. \lim_{x \rightarrow 2} \frac{x-2}{\ln(x-1)}$$

# Solutions

Although these were all taken from the textbook's answer key, there may still be typos.

1.
  - a.  $\frac{1}{32}, \frac{1}{64}$
  - b.  $a_1 = 1, a_n + 1 = \frac{a_n}{2}$
  - c.  $a_n = \frac{1}{2^{n-1}}$
2.
  - a.  $-6, 7$
  - b.  $a_1 = 1, a_{n+1} = (-1)^n(|a_n| + 1)$
  - c.  $a_n = (-1)^{n+1}n$
3.
  - a.  $32, 64$
  - b.  $a_1 = 1, a_{n+1} = 2a_n$
  - c.  $a_n = 2^{n-1}$
4.
  - a.  $243, 729$
  - b.  $a_1 = 1, a_{n+1} = 3a_n$
  - c.  $a_n = 3^{n-1}$
5.  $2e$
6.  $1$
7.  $0$ , not monotone
8. diverges
9.  $\frac{1093}{2916}$
10.  $\frac{5}{2}$
11.  $\frac{-1}{171}$
12.  $\frac{\pi}{4}$
13. Divergent by Divergence Test.
14. Converges by Integral Test.
15. Converges by Integral Test.
16. Converges by Integral Test.
17. Converges by Ratio Test.
18. Converges by Comparison Test ( $\frac{1}{k^{\ln(k)}} < \frac{1}{k^2}$  for  $k > 10$ ).
19. Converges by Comparison Test ( $\frac{1}{k^2}$ ).
20. Converges by Ratio Test.
21. Converges.
22. Converges.
23. Diverges by Divergence Test.
24. Converges by Comparison Test ( $\frac{\pi}{2} \frac{1}{k^3}$ ). Converges absolutely.
25.  $p_0(x) = 0$   
 $p_1(x) = -x$   
 $p_2(x) = -x - \frac{1}{2}x^2$
26.  $p_0(x) = 0$   
 $p_1(x) = x$   
 $p_2(x) = x$
27.  $p_0(x) = \frac{\sqrt{2}}{2}$   
 $p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$   
 $p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) - \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})^2$
28.  $p_0(x) = 10$   
 $p_1(x) = 10 + \frac{(x-100)}{20}$   
 $p_2(x) = 10 + \frac{(x-100)}{20} - \frac{(x-100)^2}{8000}$
29. Radius:  $\infty$   
 Interval:  $(-\infty, \infty)$
30. Radius:  $1$   
 Interval:  $(0, 2)$
31.  $g(x) = \frac{1}{6} \sum_{k=0}^{\infty} (k+1)(k+2)(k+3)x^k$   
 Interval:  $(-1, 1)$
32.  $g(x) = - \sum_{k=1}^{\infty} \frac{3^k}{k} x^k$   
 Interval:  $[-\frac{1}{3}, \frac{1}{3})$
33.
  - a.  $1 - x^2 + x^4 - x^6 + \dots$
  - b.  $\sum_{k=0}^{\infty} (-1)^k x^{2k}$
34.
  - a.  $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
  - b.  $\sum_{k=0}^{\infty} (-1)^k \frac{1}{2k+1} x^{2k+1}$
35.
  - a.  $1 - \frac{(x-\frac{\pi}{2})^2}{2} + \frac{(x-\frac{\pi}{2})^5}{24} - \frac{(x-\frac{\pi}{2})^6}{720} + \dots$
  - b.  $\sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k)!} \left(x - \frac{\pi}{2}\right)^{2k}$
36.
  - a.  $\ln(4) + \frac{(x-3)}{3} - \frac{(x-3)^2}{18} + \frac{(x-3)^3}{81} + \dots$
  - b.  $\ln(3) + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k3^k} (x-3)^k$
37.  $2$
38.  $\frac{2}{5}$
39.  $-\frac{1}{6}$
40.  $1$