MAT271 Exam 2 Review

Spring 2014

This is not a complete list of topics covered in class, but merely a compilation of supplemental exercises from each section. You should still review class notes and the practice exam posted on ASU's MAT271 course page:
`http://math.asu.edu/first-year-math/mat-271-calculus-analytic-geometry-ii`

§6.1 Velocity and Net Change

Assume the following velocities have units of m/s and let t be measured in seconds. Find both the displacement and distance traveled over the given intervals.

1. $v(t) = 6 - 2t$; $0 \leq t \leq 6$

2. $v(t) = t^3 - 5t^2 + 6t$; $0 \leq t \leq 5$

Find the position and velocity of an object moving along a straight line with the given acceleration, initial velocity, and position. Assume units of meters and seconds.

3. $a(t) = -9.8$, $v(0) = 20$, $s(0) = 0$

4. $a(t) = -0.01t$, $v(0) = 10$, $s(0) = 0$

§6.2 Regions Between Curves

Sketch the region bounded by the graphs of the functions, and find the area of the region.

5. $f(x) = x^2 + 2x + 1$, $g(x) = 3x + 3$

6. $f(x) = \dfrac{10}{x}$, $x = 0$, $y = 2$, $y = 10$

7. $f(x) = 2\sin(x)$, $g(x) = \tan(x)$, $-\dfrac{\pi}{3} \leq x \leq \dfrac{\pi}{3}$

8. $f(x) = \cos(x)$, $g(x) = 2 - \cos(x)$, $0 \leq x \leq 2\pi$

§6.3 Volume by Slicing

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

9. $y = \sqrt{x}$, $y = 0$, $x = 4$, the line $x = 0$

10. $y = \dfrac{1}{1+x}$, $y = 0$, $x = 0$, $x = 3$ the line $y = 4$

11. $y = x^2$, $y = 4x - x^2$, the line $y = 0$

12. $y = \sin(x)$, $y = 0$, $x = 0$, $x = \pi$ the line $y = 0$

§6.4 Volume by Shells

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

13. $y = 4 - x$, $y = 2$, $x = 0$, the line $y = 0$

14. $y = \dfrac{1}{x+1}$, $y = 1 - \dfrac{x}{3}$, the line $y = 0$

15. $y = x^2$, $x = 1$, $y = 0$, the line $x = -2$

16. $y = x$, $y = 2x + 2$, $x = 2$, $x = 2$, $x = 6$, the line $x = 0$

§6.5 Length of Curves

Find the arc length of the following curves on the given interval by integrating with respect to x.

17. $y = \dfrac{(x^2+2)^{3/2}}{3}$; $[0, 1]$

18. $y = \ln(\cos(x))$; $[0, \frac{\pi}{4}]$

19. $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$; $[1, 2]$

20. $y = 75\left(e^{x/150} + e^{-x/150}\right)$; $[-100, 100]$

§6.6 Physical Applications

21. It takes 100 J of work to stretch a spring 0.5 m from its equilibrium position. How much work is needed to stretch it an additional 0.75 m?

22. A water tank is shaped like an inverted cone with height 6 m and a base radius 1.5 m. If the tank is full, how much work is required to pump the water to the level of the top of the tank and out of the tank?

23. A swimming pool has the shape of a box with a base that measures 25 m by 15 m and a depth of 2.5 m. How much work is required to pump the water out of the pool when it is full?

24. A water trough has a semicircular cross section with a radius of 0.25 m and a length of 3 m. How much work is required to pump the water out of the trough when it is full?

Solutions

Although these were all taken from the textbook's answer key, there may still be typos.

1. displacement: 0, distance: $18\,\mathrm{m}$
2. displacement: $\frac{275}{12}\,\mathrm{m}$, distance: $23.75\,\mathrm{m}$
3. $s(t) = -4.9t^2 + 20t\,\mathrm{m}$; $v(t) = -9.8t + 20\,\mathrm{m/s}$
4. $s(t) = -\frac{0.005}{3}t^3 + 10t\,\mathrm{m}$; $v(t) = -0.005t^2 + 10\,\mathrm{m/s}$
5. $\frac{9}{2}$
6. $10\ln(5) \approx 16.094$
7. $2[1 - \ln(2)] \approx 0.614$
8. $4\pi \approx 12.566$
9. $\frac{128\pi}{5}$
10. $\pi[16 - \ln(2) - \frac{3}{4}] \approx 32.485$
11. $\frac{32\pi}{3}$
12. $\frac{\pi^2}{2} \approx 4.935$
13. $\frac{32\pi}{3}$
14. $\frac{8\pi}{27}$
15. $\frac{11\pi}{6}$
16. $\frac{608\pi}{3}$
17. $\frac{4}{3}$
18. $\ln(\sqrt{2}+1) \approx 0.881$
19. $\frac{123}{32}$
20. $150(e^{2/3} - e^{-2/3}) \approx 215$
21. $525\,\mathrm{J}$
22. $66,150\pi\,\mathrm{J}$
23. $11,484,375\,\mathrm{J}$
24. $\approx 32,667\,\mathrm{J}$