Recitation 13: Taylor Series and Parametric Equations

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November 14, 2014

Example (Rec Ntbk §9.3, #3).

a. Find the first four nonzero terms of the Taylor series centered at 2 for the function $f(x) = \frac{1}{x}$.

b. Write the power series using summation notation.

$$p_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$$

In summation notation, this is

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{k+1}}(x-2)^k$$

Example (Rec Ntbk §9.4, #1)**.** Evaluate the limit using the Taylor series: $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$.

Use the substitution $x = \frac{1}{t}$ and note that $x \to \infty$ as $t \to 0^+$. Also, let's pretend we know nothing about the sinc function or L'Hopital's and approach this purely with a Taylor series. (Aside: $\operatorname{sinc}(x) = \frac{\sin(x)}{x}$, and we call it the "sinc" or "cardinal sine" function.)

$$
\begin{aligned}
\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) &= \lim_{t\to 0^+} \frac{\sin(t)}{t} \\
&= \lim_{t\to 0^+} \frac{\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} + \cdots\right)}{t} \\
&= \lim_{t\to 0^+} \left(1 - \frac{t^2}{3!} + \frac{t^4}{5!} + \cdots\right) \\
&= 1.
\end{aligned}
$$

Assignment

Recitation Notebook:
§9.3 - #1, #2, #4
§9.4 - #2, #3, #4

As always, you may work in groups, but every member must individually submit a homework assignment.