Recitation 11: Alternating Series

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Theorem (Alternating Series Test). *The alternating series* $\sum(-1)^k a_k$ *converges if*

- $a_k \geq a_{k+1} > 0$ *for all* $k > N$, *where* N *is sufficiently large, and*
- $\lim_{k\to\infty} a_k = 0$.

Definition. Given a series $\sum a_n$, we say that $\sum a_n$ *converges absolutely* if $\sum |a_n|$ converges. We say that it *converges conditionally* if $\sum a_n$ converges but $\sum |a_n|$ does not.

Example. Determine whether the following series converges. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2+4}}$

Solution.

Since $\frac{1}{\sqrt{k^2+4}} \geq \frac{1}{\sqrt{(k+1)^2+4}}$ for ever $k \geq 0$ and $\displaystyle\lim_{k\to\infty} \frac{1}{\sqrt{k^2+4}} = 0$, the series converges by the alternating series test.

Example. Determine whether the following series converge absolutely or conditionally.

$$\sum_{k=1}^{\infty} \frac{(-1)^k \arctan(k)}{k^3}$$

Solution.

It's not hard to see that $\frac{\arctan(k)}{k^3} \geq \frac{\arctan(k+1)}{k^3}$ for some sufficiently large k (try taking a derivative of $\frac{\arctan(x)}{x^3}$ and see that it is negative for the interval $(0, \infty)$). As well, $\lim_{k\to\infty} \frac{\arctan(k)}{k^3} = 0$, so the series converges by the Alternating Series Test.

Since $0 \leq \arctan(k) \leq \frac{\pi}{2}$ for every $k \geq 1$, we can use the comparison test with the convergent p-series $\sum_{k=1}^{\infty} \frac{pi}{2}\left(\frac{1}{k^3}\right)$ to see that $\sum_{k=1}^{\infty} \left|\frac{(-1)^k \arctan(k)}{k^3}\right| = \sum_{k=1}^{\infty} \frac{\arctan(k)}{k^3}$ converges.

Assignment

Recitation Notebook:
§8.5 - #1(b), #2, #3, #4
§8.6 - #1(b), #2, #6

As always, you may work in groups, but every member must individually submit a homework assignment.