Recitation 01: Introduction & Integration Recap

Joseph Wells
Arizona State University

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For the policies, lectures notes, and homework assignments, please visit my website:

`http://math.joedub.net`

Example. True or False:

a. If f is integrable on the domain $[a,b]$, then $\displaystyle\int_a^b f'(t)\,dt = f(b) - f(a)$

b. If $A(x) = \displaystyle\int_a^x f(t)\,dt$ and $f(t) = 2t - 3$, the A is a quadratic function.

c. If the average value of f on $[a,b]$ is 0, then $f(x) = 0$.

d. $\displaystyle\int_a^b [2f(t) - 3g(t)]\,dt = 2t\int_a^b f(t)\,dt - 3t\int_a^b g(t)\,dt.$

Solution.

a. True. This is the fundamental theorem of calculus!

b. True. By the fundamental theorem of calculus, $A(x) = t^2 - 3t\Big|_a^x = x^2 - 3x - a^2 + 3a$.

c. False. Consider $f(x) = x$ on $[-1, 1]$. The function is nonzero, but the average value is $\frac{1}{1-(-1)} \int_{-1}^{1} x\, dx = 0$.

d. False. It should be $\displaystyle\int_a^b [2f(t) - 3g(t)]\, dt = 2\int_a^b f(t)\, dt - 3\int_a^b g(t)\, dt$.

Example. Evaluate the following integrals:

a. $\displaystyle\int (3x^4 - 2x + 1)\,dx$

b. $\displaystyle\int \cos(3x)\,dx$

c. $\displaystyle\int e^{4x+8}\,dx$

d. $\displaystyle\int_0^1 (8x^5 + x^2 + 1)\,dx$

e. $\displaystyle\int_{-\pi}^{\pi/2} \cos(x)\,dx$

f. $\displaystyle\int_0^{\ln(2)} 5e^{2x}\,dx$

Solution.

a. $\frac{3}{5}x^5 - x^2 + x + C$

b. $\frac{1}{3}\sin(3x) + C$

c. $\frac{1}{4}e^{4x+8} + C$

d. $\frac{8}{6}x^6 + \frac{1}{3}x^3 + x\Big|_0^1 = \frac{8}{3}$

e. $\int_{-\pi}^{\pi/2} \sin(x)\,dx = -\cos(x)\Big|_{-\pi}^{\pi/2} = -1$

f. $\frac{5}{2}e^{2x}\Big|_0^{\ln(2)} = \frac{15}{2}$

Theorem. *(Substitution Rule) Let $u = g(x)$, where g' is continuous on an $[a, b]$, and let f be continuous on the range of g. Then*

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Example. Evaluate the following integrals using a u-subtitution:

a. $\displaystyle\int \frac{x^2+2x-2}{x^3+3x^2-6x}\,dx$

b. $\displaystyle\int_0^{\ln(2)} \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

c. $\displaystyle\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)\,dx$

Solution

a. Let $u = x^3+3x^2-6x$. Then $du = (3x^2+6x-6)\,dx$, so

$$\begin{aligned}\int \frac{x^2+2x-2}{x^3+3x^2-6x}\,dx &= \frac{1}{3}\int \frac{du}{u}\\ &= \frac{1}{3}\ln|u|+C\\ &= \frac{1}{3}\ln|x^3+3x^2-6x|+C.\end{aligned}$$

b. Let $u = e^x + e^{-x}$. Then $du = (e^x - e^{-x})\,dx$, so

$$\begin{aligned}\int_0^{\ln(2)} \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx &= \int_2^{5/2} \frac{du}{u} \\ &= \ln(u)\Big|_2^{5/2} \\ &= \ln(5/4)\end{aligned}$$

c. Let $u = \frac{1}{x}$. Then $du = -\frac{dx}{x^2}$, so

$$\begin{aligned}\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)\,dx &= \int -\sin(u)\,du \\ &= \cos(u) + C \\ &= \cos\left(\frac{1}{x}\right)\end{aligned}$$

Assignment

Worksheet 01:

http://math.joedub.net/teaching/mat271_fall2014/homework01.pdf