

Section 4.3

1. THIS ONE'S ON YOU. 😊

3. CRITICAL POINTS AT $x = -4, -2, \text{ AND } 1$, SO
WE'LL TEST $-5, -3, 0, 2$.

$$f(-5) < 0,$$

$(-\infty, -4)$ DECREASING

$$f(-3) > 0,$$

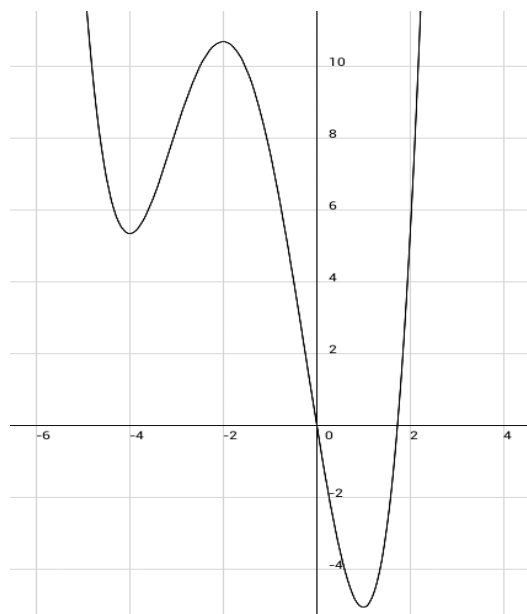
$(-4, -2)$ INCREASING

$$f(0) < 0,$$

$(-2, 1)$ DECREASING

$$f(2) > 0,$$

$(1, \infty)$ INCREASING



4. $f'(1) = 0$

CRITICAL POINT AT $x = 1$

$$f'(x) > 0 \text{ FOR } x < 1,$$

$(-\infty, 1)$ INCREASING

$$f'(x) > 0 \text{ FOR } x > 1,$$

$(1, \infty)$ INCREASING

$$f''(1) = 0$$

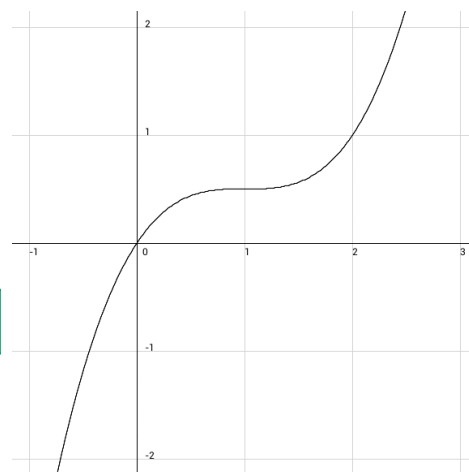
INFLECTION POINT AT $x = 1$.

$$f''(x) < 0 \text{ FOR } x < 1,$$

$(-\infty, 1)$ CONCAVE DOWN

$$f''(x) > 0 \text{ FOR } x > 1,$$

$(1, \infty)$ CONCAVE UP



Section 4.4

2. MAXIMIZE: $V = b^2 h$ CONSTRAINT: $108 \text{ cm} = 2b + h$
 $\Rightarrow V(b) = b^2(108 - 2b)$
 $\quad = 108b^2 - 2b^3$

$$0 = \frac{dV}{db} = 216b - 6b^2 = 6b(36 - b)$$

$$\begin{aligned} \rightarrow 108 &= 2(36) + h \\ \Rightarrow h &= 36 \text{ in} \end{aligned}$$

SINCE $b = 0$ in DOESN'T MAKE SENSE, $b = 36$ in

DIMENSIONS: $36 \text{ in} \times 36 \text{ in} \times 36 \text{ in}$
 VOLUME: $46,656 \text{ in}^3$

4. $x =$ SQUARE SIDE LENGTH, $r =$ CIRCLE RADIUS $0 \leq x \leq 15$

OPTIMIZE: $A = x^2 + \pi r^2$ CONSTRAINT: $60 = 4x + 2\pi r$
 $\Rightarrow A = \left(\frac{60 - 2\pi r}{4}\right)^2 + \pi r^2$

$$0 = \frac{dA}{dr} = \frac{\pi}{2}(4 + \pi)r - 15\pi$$

\Rightarrow CRITICAL POINT AT

$$r = \frac{30}{4 + \pi} \text{ cm}$$

NOTICE THAT $r \in [0, \frac{30}{\pi}]$ AND $0 < \frac{30}{4 + \pi} < \frac{30}{\pi}$

SO WE PLUG IN FOR CRITICAL AND END POINTS.

a. $A|_{r=0} = 225 \text{ cm}^2$

$A|_{r=\frac{30}{4+\pi}} \approx 126.02 \text{ cm}^2$

$\frac{60\pi}{4+\pi}$ cm FOR CIRCLE, $\frac{240}{4+\pi}$ cm FOR SQUARE.

b. $A|_{r=\frac{30}{\pi}} \approx 286.48 \text{ cm}^2$

60 cm FOR CIRCLE, 0 cm FOR SQUARE.

