Section 3.7

1.

a. $\frac{d}{dx}\left[\tan(xy)\right] = \frac{d}{dx}\left[x+y\right]$

$\frac{d}{dx}[xy]\sec^2(xy) = 1 + y'$

$(y + xy')\sec^2(xy) = 1 + y'$

$y\sec^2(xy) - 1 = y'(1 - x\sec^2(xy))$

$$\boxed{y' = \frac{y\sec^2(xy) - 1}{1 - x\sec^2(xy)}}$$

b. $y'\Big|_{(0,0)} = \frac{0-1}{1-0} = \boxed{-1}$

2. $\frac{d}{dx}\left[(xy+1)^3\right] = \frac{d}{dx}\left[x - y^2 + 8\right]$

$3\frac{d}{dx}[xy+1](xy+1)^2 = 1 - 2yy'$

$3(y + xy')(xy+1)^2 = 1 - 2yy'$

$3y(xy+1)^2 + 3xy'(xy+1)^2 = 1 - 2yy'$

$3xy'(xy+1)^2 + 2yy' = 1 - 3y(xy+1)^2$

$y'(3x(xy+1)^2 + 2y) = 1 - 3y(xy+1)^2$

$$\boxed{y' = \frac{1 - 3y(xy+1)^2}{3x(xy+1)^2 + 2y}}$$

3.

a. $(1)^3+(1)^3 \stackrel{?}{=} 2(1)(1)$

$2=2$ $\boxed{\text{YES, POINT IS ON CURVE}}$

b. $\frac{d}{dx}\left[x^3+y^3\right]=\frac{d}{dx}\left[2xy\right]$

$3x^2+3y^2y' = 2y+2xy'$

$3y^2y'-2xy'=2y-3x^2$

$y'(3y^2-2x)=2y-3x^2$

$y'=\frac{2y-3x^2}{3y^2-2x}$

$m=y'\big|_{(1,1)}=\frac{2-3}{3-2}=-1$

$y-1=(-1)(x-1)$

$\boxed{y=-x+2}$

4. $\frac{d}{dx}\left[x^{2/3}+y^{2/3}\right]=\frac{d}{dx}\left[2\right]$

$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}y'=0$

$y'=-\left(\frac{y}{x}\right)^{1/3}$

$\boxed{y'\big|_{(1,1)}=-1}$

Section 3.8

2.

a. $\frac{d}{dx}\left[e^x \ln(x)\right] = \boxed{e^x \ln(x) + \frac{e^x}{x}}$

$\boxed{\text{DOMAIN: } (0, \infty)}$

b. $\frac{d}{dx}\left[4^{-x} \sin(x)\right] = \frac{d}{dx}\left[4^{-x}\right]\sin(x) + 4^{-x}\cos(x)$

$= \boxed{4^{-x}\ln(4)\sin(x) + 4^{-x}\cos(x)}$

$\boxed{\text{DOMAIN: } (-\infty, \infty)}$

3. $\frac{d}{dx}\left[\log_2(\log_2 x)\right] = \frac{d}{dx}\left[\log_2(x)\right]\left(\frac{1}{\ln(2)\log_2(x)}\right)$

$= \frac{1}{\ln(2)\,x}\left(\frac{1}{\ln(2)\ \log_2(x)}\right)$

$= \frac{1}{\ln(2)\,x}\left(\frac{1}{\ln(2)}\right)\left(\frac{\ln(2)}{\ln(x)}\right)$

$= \boxed{\frac{1}{x\ln(2)\ln(x)}}$

4. $f'(x) = \frac{x^8\cos^3(x)}{\sqrt{x-1}}\frac{d}{dx}\left[\ln\left(\frac{x^8\cos^3(x)}{\sqrt{x-1}}\right)\right]$

$= \frac{x^8\cos^3(x)}{\sqrt{x-1}}\frac{d}{dx}\left[8\ln(x) + 3\ln(\cos(x)) - \frac{1}{2}\ln(x-1)\right]$

$= \boxed{\frac{x^8\cos^3(x)}{\sqrt{x-1}}\left(\frac{8}{x} - \frac{3\sin(x)}{\cos(x)} - \frac{1}{2(x-1)}\right)}$