Recitation 13: Antiderivatives & Integrals Galore

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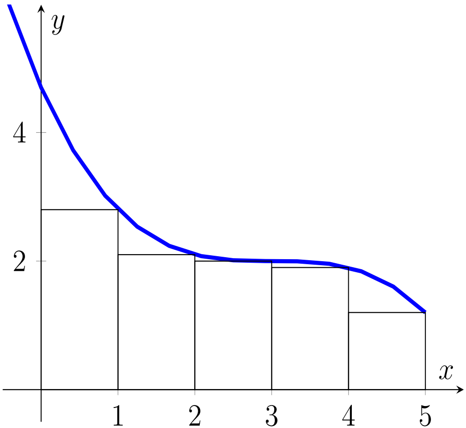
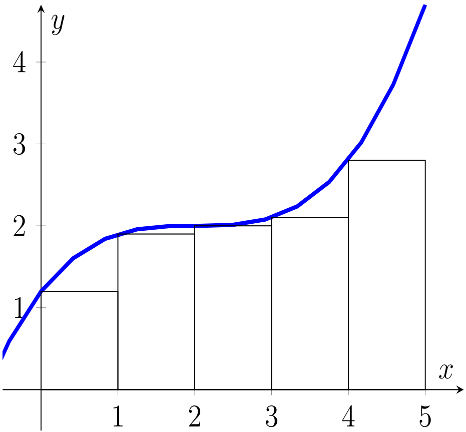
Example (§5.1, #1)**.**

a. Does the right Riemann sum underestimate or overestimate the area of the region under the graph of a positive decreasing function? Explain.

b. Does the left Riemann sum underestimate or overestimate the area of the region under the graph of a positive increasing function?

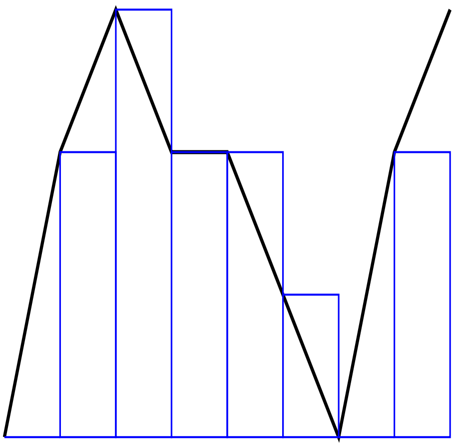


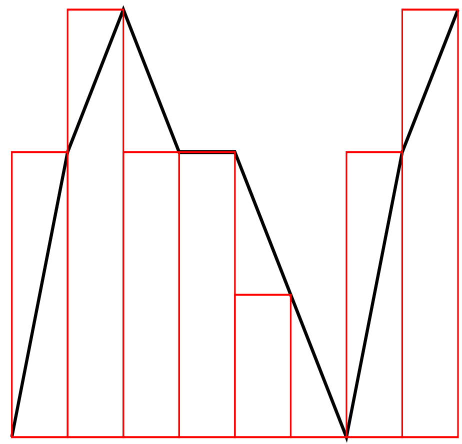


Example (§5.1, #3)**.** Use the tabulated values of f to evaluate the left and right Riemann sums for $n = 8$ in the interval $[1, 5]$.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(x)$	0	2	3	2	2	1	0	2	3

Solution.





So $L = (0.5)(0+2+3+2+2+1+0+2) = 6$ and $R = (0.5)(2+3+2+2+1+0+2+3) = \frac{15}{2}$.

Example (§5.2, #4)**.**

a. Use geometry to find a formula for $\int_0^a x\,dx$ in terms of a.

b. If f is integrable and $\int_a^b |f(x)|\,dx = 0$, what can you conclude about f?

Solution.

a. Notice that, for $0 \le x \le a$, the area under $f(x)$ is a triangle with base a and height a. The area of the triangle is thus $\frac{1}{2}a^2$.

b. We can conclude that $f(x) = 0$.

Example (§5.3, #1)**.** Evaluate the following integral using the Fundamental Theorem of Calculus. Discuss whether your result is consistent with the figure. $\displaystyle\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\, dx$

Solution.

We solve this in the usual fashion:

$$\begin{aligned}
\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\, dx &= -\cos x + \sin x\Big|_{-\pi/4}^{7\pi/4} \\
&= \left[-\cos\left(\frac{7\pi}{4}\right) + \sin\left(\frac{7\pi}{4}\right)\right] - \left[-\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right] \\
&= \left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] - \left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] \\
&= 0.
\end{aligned}$$

Example (§5.3, #3). Simplify the expression: $\dfrac{d}{dx}\displaystyle\int_{x^2}^{10} \frac{dz}{z^2+1}$.

Solution.

By the Fundamental Theorem of Calculus,

$$\begin{aligned} \frac{d}{dx}\left[\int_{x^2}^{10} f(z)\,dz\right] &= \frac{d}{dx}\left[F(10) - F(x^2)\right] \\ &= -2xf(x^2), \qquad \text{(chain rule)} \end{aligned}$$

so in particular,

$$\frac{d}{dx}\int_{x^2}^{10} \frac{dz}{z^2+1} = -\frac{2x}{x^4+1}.$$

As justification that this is correct, we can brute force our way through it (although in general, it's rare that we could compute the antiderivative so easily, if even at all.)

$$
\begin{aligned}
\frac{d}{dx}\int_{x^2}^{10}\frac{dz}{z^2+1} &= \frac{d}{dx}\left[\arctan(z)\Big|_{x^2}^{10}\right] \\
&= \frac{d}{dx}\left[\arctan(10) - \arctan(x^2)\right] \\
&= -\frac{2x}{x^4+1}.
\end{aligned}
$$

Example (§5.4, #3)**.** Use symmetry to evaluate the following integrals. Draw a figure to interpret your result.

a. $\displaystyle\int_0^{2\pi} \cos x\, dx$

b. $\displaystyle\int_0^{2\pi} \sin x\, dx$

Solution.

a. In our interval, $\cos x$ is symmetric about $x = \pi$. So we can rewrite the integral as $2\int_0^{\pi} \cos x\, dx$. Looking at it again, we see that in our new interval, $\cos x$ is antisymmetric about $\frac{\pi}{2}$ meaning $\int_0^{\pi/2} \cos x\, dx = -\int_{\pi/2}^{\pi} \cos x\, dx$. Thus

$$\begin{aligned}\int_0^{2\pi} \cos x \, dx &= 2\int_0^{\pi} \cos x \, dx \\ &= 2\int_0^{\pi/2} \cos x \, dx + 2\int_{\pi/2}^{\pi} \cos x \, dx \\ &= 2\int_0^{\pi/2} \cos x \, dx - 2\int_0^{\pi/2} \cos x \, dx \\ &= 0.\end{aligned}$$

b. In our interval, $\sin x$ is antisymmetric about $x = \pi$, so $\int_0^{\pi} \sin x \, dx = -\int_{\pi}^{2\pi} \cos x \, dx$. Thus

$$\begin{aligned}\int_0^{2\pi} \sin x \, dx &= \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} \sin x \, dx \\ &= \int_0^{\pi} \sin x \, dx - \int_0^{\pi} \sin x \, dx \\ &= 0.\end{aligned}$$

Assignment

MAT270 Recitation Notebook
§5.1, Problems 2
§5.2, Problems 1
§5.3, Problems 2,4
§5.4, Problems 2