Recitation 10: Optimization

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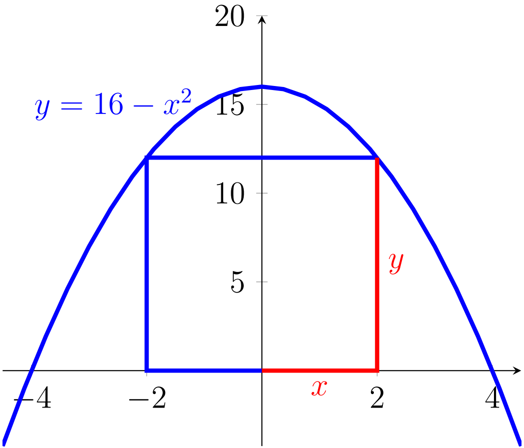
Example (§4.3, #18 (in the book))**.** A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 16 - x^2$. What are the dimensions of the rectangle with the maximum area? What is the area?

Solution.

Let's begin by looking at a picture



It's important to see that x is only half of the base of the rectangle. This will affect our equations.

$$\text{Optimize}: \ A = bh = (2x)y$$
$$\text{Constraint}: \ y = 16 - x^2$$
$$\text{Interval of } x: \ (0, 4)$$
$$\text{Interval of } y: \ (0, 16)$$

We plug in our constraint to our objective function (the function we're optimizing)

$$\begin{aligned} A &= 2xy \\ &= 2x(16 - x^2) \\ &= 32x - 2x^3 \end{aligned}$$

and since A is a function of x, we look for our (local) extrema on the interval $(0, 4)$.

$$\begin{aligned} 0 &= \frac{dA}{dx} \\ 0 &= 32 - 6x^2 \\ \Rightarrow x &= \pm\frac{4}{\sqrt{3}} \end{aligned}$$

so we have a critical point at $x = \dfrac{4}{\sqrt{3}}$. A quick check with the first derivative test tells us that our function A is incincreasing on $\left(0, \frac{4}{\sqrt{3}}\right)$ and decreasing on $\left(\frac{4}{\sqrt{3}}, 4\right)$, so $\dfrac{4}{\sqrt{3}}$ corresponds to a local maximum. Plugging back in, we get that our rectangle has dimensions $\frac{8}{\sqrt{3}} \times \frac{32}{3}$ and it has area $\dfrac{256}{3\sqrt{3}} \approx 49.268$.

Assignment

MAT270 Recitation Notebook
§4.3, Problems 1,3,4
§4.4, Problems 2,4