# Recitation 06: Trig Limits and Derivatives

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This week, we covered a few more limits, derivatives of trig functions, and the powerful chain rule.

**Theorem** (3.11)**.**

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

**Theorem** (3.12)**.**

$$\frac{d}{dx}[\sin(x)] = \cos(x) \qquad \frac{d}{dx}[\cos(x)] = -\sin(x)$$

**Theorem** (3.13)**.**

$$\frac{d}{dx}[\sin(x)] = \cos(x) \qquad \frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$
$$\frac{d}{dx}[\cos(x)] = -\sin(x) \qquad \frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$
$$\frac{d}{dx}[\tan(x)] = \sec^2(x) \qquad \frac{d}{dx}[\cot(x)] = -\csc^2(x)$$

**Theorem** (3.14 - Chain Rule). *Suppose $g$ is differentiable at $x$ and $y = f(u)$ is differentiable at $u = g(x)$. The composite function $y = f(g(x))$ is differentiable at $x$, and its derivative can be expressed in two ways:*

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

**Theorem** (3.15 - Chain Rule (for Powers)). *If $g$ is differentiable for all $x$ in the domain and $n$ is an integer, then*

$$\frac{d}{dx}[(g(x))^n] = n(g(x))^{n-1} \cdot g'(x).$$

**Example** (§3.4, #3)**.** Evaluate the following limits or state that they do not exist.

$$\lim_{x\to 0} \frac{\sin(ax)}{bx}, \quad \text{where } a \text{ and } b \text{ are constants with } b \neq 0.$$

*Solution.*

$$\lim_{x\to 0} \frac{\sin(ax)}{bx} = \frac{1}{b}\left[\lim_{x\to 0} \frac{\sin(ax)}{x}\right] = \frac{1}{b}\left[\lim_{x\to 0} \frac{a\sin(ax)}{ax}\right] = \frac{a}{b}\left[\lim_{x\to 0} \frac{\sin(ax)}{ax}\right]$$

substituting $t = ax$,

$$= \frac{a}{b}\left[\lim_{t\to 0} \frac{\sin(t)}{t}\right]$$

and applying Theorem 3.11

$$= \frac{a}{b} \cdot 1 = \frac{a}{b}$$

**Example** (§3.6 #4)**.** Suppose $f$ is differentiable on $[-2, 2]$ with $f'(0) = 3$ and $f'(1) = 5$. Let $g(x) = f(\sin(x))$. Evaluate the following expressions.

a $g'(0)$

b $g'\left(\frac{\pi}{2}\right)$

c $g'(\pi)$

*Solution.*

First thing's first - let's figure out the derivative of $g(x)$

$$\frac{d}{dx}g(x) = \frac{d}{dx}f(\sin(x)) = [\sin(x)]' \cdot f'(\sin(x)) = \cos(x) \cdot f'(\sin(x))$$

a

$$g'(0) = \cos(0)f'(\sin(0)) = 1 \cdot f'(0) = 3$$

b

$$g'\left(\frac{\pi}{2}\right) = \cos\left(\frac{pi}{2}\right) \cdot f'\left(\sin\left(\frac{pi}{2}\right)\right) = 0 \cdot f'(1) = 0$$

c

$$g'(\pi) = \cos(\pi) \cdot f'(\sin(\pi)) = -1 \cdot f'(0) = -3$$

## Assignment

MAT270 Recitation Notebook
§3.4, Problems 1,2,4
§3.6, Problems 1,2,3