Recitation 05: Derivatives

Joseph Wells
Arizona State University

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Rules of Differentiation

Constant: $$\frac{d}{dx}(c) = 0$$

Power: $$\frac{d}{dx}(x^n) = nx^{n-1}$$

Constant Multiple: $$\frac{d}{dx}[cf(x)] = cf'(x)$$

Exponential: $$\frac{d}{dx}(e^x) = e^x$$

Constant Exponential: $$\frac{d}{dx}(e^{kx}) = ke^{kx}$$

Sum: $$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

Product: $$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient: $$\frac{d}{dx}\left[\frac{d}{dx}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

The derivative of a function $f(x)$ is another function, $f'(x)$, that represents the slope at every point in our original function $f(x)$. If we need to know the slope at a particular point $x = a$, we simply compute the derivative and then plug in the x-value. This process would be denoted $f'(a)$ or, somewhat less commonly, $\left.\dfrac{d}{dx} f(x)\right|_{x=a}$.

Example (§3.2, #4). Let $F = f + g$ and $G = 3f - g$, where the graphs of f and g are shown in the figure (in the recitation notebook). Find the following derivatives.

a. F'(1)

b. G'(5)

Solution.

a. Since $F = f + g$, then $F' = f' + g'$. At $x = 1$, the slope of f is -3 and the slope of g is 1. Therefore

$$F'(1) = f'(1) + g'(1) = -3 + 1 = -2.$$

b. Since $G = 3f - g$, then $G' = 3f' - g'$. At $x = 5$, the slope of f is 1 and the slope of g is -1. Therefore

$$G'(5) = 3f'(5) - g'(5) = 3(1) - (-1) = 4.$$

Example (§3.3, #4)**.** Suppose the derivative of f exists, and assume that $f(2) = 2$ and $f'(2) = 3$. Let $g(x) = x^2 \cdot f(x)$ and $h(x) = \dfrac{f(x)}{x-3}$.

a. Find an equation of the line tangent to $y = g(x)$ at $x = 2$.

b. Find an equation of the line tangent to $y = h(x)$ at $x = 2$.

Solution.

a. We first start by finding the slope at $x = 2$, given by $g'(2)$. So first we differentiate $g(x)$

$$g'(x) = [x^2]' f(x) + x^2 f'(x) = 2x f(x) + x^2 f'(x),$$

and then we plug in $x = 2$

$$m = g'(2) = 2(2) f(2) + (2)^2 f'(2) = 4(2) + 4(3) = 20.$$

Now that we have our slope, we can use our point-slope form to determine the equation of the tangent line passing through the point $(2, g(2)) = (2, 8)$. I'll call this equation t:

$$\begin{aligned} t-8 &= 20(x-2) \\ t-8 &= 20x-40 \\ t &= 20x-32. \end{aligned}$$

b. We repeat the same process here for $h(x)$.

$$h'(x) = \frac{f(x)}{x-3} = \frac{f'(x)(x-3) - f(x)[x-3]'}{(x-3)^2} = \frac{f'(x)(x-3) - f(x)}{(x-3)^2}$$

so our slope at $x = 2$ is

$$m = h'(2) = \frac{f'(2)(2-3) - f(2)}{(2-3)^2} = \frac{3(-1) - 2}{1} = -5.$$

The equation of our tangent line t going through the point $(2, h(2)) = (2, -2)$ is thus

$$\begin{aligned} t-(-2) &= -5(x-2) \\ t+2 &= -5x+10 \\ t &= -5x+8. \end{aligned}$$

Example (§3.3, #5)**.** Use the following table to find the given derivative.

x	1	2	3	4	5
$f(x)$	5	4	3	2	1
$f'(x)$	3	5	2	1	4
$g(x)$	4	2	5	3	1
$g'(x)$	2	4	3	1	2

$$\frac{d}{dx}[f(x)g(x)]\bigg|_{x=1}$$

Solution.

We first find the derivative

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x),$$

and then we plug in $x = 1$ and reference the table to plug in the values

$$f'(1)g(1) + f(1)g'(1) = 3(4) + 5(2) = 22.$$

Assignment

MAT270 Recitation Notebook
§3.2, Problems 1,2,3
§3.3, Problems 1,3