1. We have that

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right| = \lim_{n\to\infty}\left|\frac{x}{n+1}\right| = 0$$

So, by the ratio test, this converges for all x-values. Thus the radius of convergence is $\boxed{R=\infty}$ and the interval of convergence is $\boxed{(-\infty,\infty)}$.

2. We have that

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)^2+1}\cdot\frac{n^2+1}{(x-2)^n}\right| = \lim_{n\to\infty}\left|(x-2)\cdot\frac{(n+1)^2+1}{n^2+1}\right| = |x-2|.$$

By the Ratio Test, this converges when $|x-2|<1$, and so the radius of convergence is $\boxed{R=1}$. The open interval of convergence is $(1,3)$. When $x=1$, this series converges by the Alternating Series Test. When $x=3$, this series converges by comparison with $\sum\frac{1}{n^2}$. Thus the interval of convergence is $\boxed{[1,3]}$.

3. We have that

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)3^{n+1}}\cdot\frac{n3^n}{x^n}\right| = \lim_{n\to\infty}\left|\frac{x}{3}\cdot\frac{n}{n+1}\right| = \left|\frac{x}{3}\right|.$$

By the Ratio Test, this converges when $\left|\frac{x}{3}\right|<1$, or rather, when $|x|<3$. Thus the radius of convergence is $\boxed{R=3}$ and the open interval of convergence is $(-3,3)$. When $x=-3$ the series converges by the Alternating Series Test, and when $x=3$ it diverges since it is the harmonic series. Thus the interval of convergence is $\boxed{[-3,3)}$.

4. We see by the divergence test that the series diverges for all $x\neq\frac{1}{2}$. However, when $x=\frac{1}{2}$, we are effectively summing up 0 infinitely many times, and so the series converges only at $x=\frac{1}{2}$. As such, we write that the radius of convergence is $\boxed{R=0}$ and the interval of convergence is $\boxed{\left[\frac{1}{2},\frac{1}{2}\right], \text{ or } \left\{\frac{1}{2}\right\}.}$

5.

6. Given the usual power series for $1/(1-x)$, we get

$$\begin{aligned} f(x) = \frac{1}{1+x} &= \frac{1}{1-(-x)} \\ &= \sum_{n=0}^{\infty}(-x)^n \\ &= \boxed{\sum_{n=0}^{\infty}(-1)^n x^n} \end{aligned}$$

The open interval of convergence is $\boxed{(-1,1).}$

7. Given the usual power series for $1/(1-x)$, we get

$$\begin{aligned} f(x) = \frac{x}{2x^2+1} &= x \cdot \left(\frac{1}{1-(-2x^2)}\right) \\ &= x \cdot \sum_{n=0}^{\infty} (-2x^2)^n \\ &= x \cdot \sum_{n=0}^{\infty} (-1)^n 2^n x^{2n} \\ &= \boxed{\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1}} \end{aligned}$$

The open interval of convergence is $\boxed{\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).}$

8.

9.

10.