MAT266 Homework 03 (Solutions)

1. Making the substitution

$$u = \sin x$$
$$du = \cos x \, dx$$

our integral becomes

$$\int \frac{\cos x}{\sin^2 x - 9} \, dx = \int \frac{1}{u^2 - 9} \, du$$

Applying the given formula, we have

$$\int \frac{1}{u^2 - 9} \, du = \frac{1}{2(3)} \ln\left|\frac{u-3}{u+3}\right| + C$$
$$= \boxed{\frac{1}{6} \ln\left|\frac{\sin x - 3}{\sin x + 3}\right| + C}$$

2. Making the substituion

$$u = \sqrt{2}y$$
$$du = \sqrt{2} \, dy$$

our integral becomes

$$\int \frac{\sqrt{2y^2 - 3}}{y^2} \, dy = \sqrt{2} \int \frac{\sqrt{u^2 - a^2}}{u^2} \, du.$$

Applying the given formula, we get

$$\sqrt{2} \int \frac{\sqrt{u^2 - a^2}}{u^2} \, du = \sqrt{2} \left(-\frac{\sqrt{u^2 - a^2}}{u} + \ln\left|u + \sqrt{u^2 - a^2}\right| \right) + C$$
$$= \sqrt{2} \left(-\frac{\sqrt{2y^2 - 3}}{\sqrt{2}y} + \ln\left|\sqrt{2}y + \sqrt{2y^2 - 3}\right| \right) + C$$
$$= \boxed{-\frac{\sqrt{2y^2 - 3}}{y} + \sqrt{2} \ln\left|\sqrt{2}y + \sqrt{2y^2 - 3}\right| + C}$$

3. We begin with integration by parts, taking

$$u = \arcsin(\sqrt{x}) \qquad du = \frac{1}{2\sqrt{x}\sqrt{1-x}} \, dx$$
$$dv = dx \qquad v = x$$

The integration by parts formula then yields

$$\int \arcsin(\sqrt{x}) \, dx = x \arcsin(\sqrt{x}) - \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} \, dx$$

Now making the substitution

$$\begin{aligned} w &= \sqrt{x} \\ dw &= \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2w\,dq \qquad\qquad\qquad\qquad = dx \end{aligned}$$

we get

$$x\arcsin(\sqrt{x}) - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx = x\arcsin(\sqrt{x}) - \int \frac{w^2}{\sqrt{1-w^2}}\,dx$$

and applying the given formula results in

$$\begin{aligned} x\arcsin(\sqrt{x}) - \int \frac{w^2}{\sqrt{1-w^2}}\,dx &= x\arcsin(\sqrt{x}) + \frac{w}{2}\sqrt{1-w^2} - \frac{1}{2}\arcsin(w) + C \\ &= \boxed{x\arcsin(\sqrt{x}) + \frac{\sqrt{x}}{2}\sqrt{1-x} - \frac{1}{2}\arcsin(\sqrt{x}) + C} \end{aligned}$$

4.

$$\begin{aligned} M_4 &= \Delta x\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + f\left(\frac{x_2+x_3}{2}\right) + f\left(\frac{x_3+x_4}{2}\right)\right] \\ &= \frac{1}{2}\left[f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right)\right] \\ &= \frac{1}{2}\left[\frac{4096}{4097} + \frac{4096}{4825} + \frac{4096}{19721} + \frac{4096}{121745}\right] \\ &= \boxed{\frac{763,041,601,200,128}{730,180,098,979,825} \approx 1.045} \end{aligned}$$

5.

$$\begin{aligned} T_4 &= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right] \\ &= \frac{\left(\frac{1}{2}\right)}{2}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right] \\ &= \frac{1}{4}\left[1 + \frac{128}{65} + 1 + \frac{128}{793} + \frac{1}{65}\right] \\ &= \boxed{\frac{16,439}{3,965} \approx 1.03651} \end{aligned}$$

6.

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\begin{aligned}
S_4 &= \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right] \\
&= \frac{\left(\frac{1}{2}\right)}{3}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + f(2)\right] \\
&= \frac{1}{6}\left[1 + \frac{256}{65} + 1 + \frac{256}{793} + \frac{1}{65}\right] \\
&= \boxed{\frac{24,887}{23,790} \approx 1.04611}
\end{aligned}
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