§8.1 Sequences

Determine whether the following sequences converge. If they converge, find the limit.

1. $\left\{\dfrac{n}{\ln n}\right\}_{n=2}^{\infty}$

2. $\left\{\dfrac{7^{n+8}}{9^n}\right\}_{n=0}^{\infty}$

§8.2 Series

Determine the convergence or divergence of the following series. If it converges, find the sum.

3. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n+1}}{3^{n-1}}$

5. $\displaystyle\sum_{n=0}^{\infty}\left[\left(\frac{2}{3}\right)^n-\frac{1}{(n+1)(n+2)}\right]$

§8.4 Other Convergence Tests

Determine whether the following series converge. Justify your answer.

6. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^n}$

7. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n^3}}$

8. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n n}{n^2-3}$

9. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^3}$

§8.5 Power Series

Find the interval and radius of convergence of the following series.

10. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{x}{10}\right)^n$

11. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{(n+1)^2}$

12. $\displaystyle\sum_{n=0}^{\infty}n!(x-2)^n$

§8.6 Representing Functions as Power Series

13. Find a power series representation for $f(x)=\frac{1}{1+x}$, centered at 0.

14. Find a power series representation for $g(x)=-\frac{1}{(1+x)^2}$, centered at 0.

15. Use power series to evaluate $\displaystyle\int\frac{3}{1-x^7}\,dx$.

§8.7 Taylor and Maclaurin Series

Find a power series representation for the following function, centered at a.

16. $f(x)=3^x$, $a=0$

17. $f(x)=\frac{1}{x}$, $a=-1$

Find the second-degree Taylor polynomial, centered at a.

18. $f(x)=e^{-x/2}$, $a=0$

19. $f(x)=\tan x$, $a=-\frac{\pi}{4}$

MAT266 Exam 03 - Review (Solutions)

1. Diverges.

2. Converges to 0.

3. $\frac{1}{2}$

4. 12

5. 2

6. Converges by ratio test.

7. Diverges by p-series test.

8. Converges by alternating series test.

9. Diverges by ratio test.

10. Radius: 10. Interval: $(-10, 10)$

11. Radius: 1. Interval: $[1, 3]$

12. Converges only at $x = 2$

13. $\displaystyle\sum_{n=0}^{\infty}(-x)^n$

14. $\displaystyle\sum_{n=1}^{\infty}(-1)^n n x^{n-1}$

15. $\displaystyle C + \sum_{n=0}^{\infty}\frac{3x^{7n+1}}{7n+1}$

16. $\displaystyle\sum_{n=0}^{\infty}\frac{(x\ln 3)^n}{n!}$

17. $\displaystyle -\sum_{n=0}^{\infty}(x+1)^n$

18. $1 - \frac{x}{2} + \frac{x^2}{8}$

19. $-1 + 2\left(x + \frac{\pi}{4}\right) - 2\left(x + \frac{\pi}{4}\right)^2$