1. **a.** Our goal is to minimize the distance from $(2,1)$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Recall the distance, z, from $(2,1)$ to a point (x,y) on the ellipse satisfies $z^2 = (x-2)^2 + (y-1)^2$. In particular, with implicit differentiation, we get

$$\frac{dz}{dx} = \frac{(x-2) + (y-1)\frac{dy}{dx}}{z} \tag{1}$$

Using implicit differentiation on our ellipse equation, we also see that

$$\frac{dy}{dx} = -\frac{9x}{25y},$$

so substituting this into Equation 1, we get

$$\frac{dz}{dx} = \frac{(x-2) + \left(-\frac{9x}{25} + \frac{9x}{25y}\right)}{z} = \frac{\frac{16x}{25} + \frac{9x}{25y} - 2}{z} \tag{2}$$

Rearranging to solve for y in the equation of the ellipse, we have

$$y = 3\sqrt{1 - \frac{x^2}{25}}$$

So, setting Equation 2 equal to 0, we see that it suffices to solve for where the numerator is 0. Hence

$$\begin{aligned}
0 &= \frac{16x}{25} + \frac{9x}{25y} - 2\\
0 &= \frac{16x}{25} + \frac{3x}{25\sqrt{1 - \frac{x^2}{25}}} - 2\\
2 - \frac{16x}{25} &= \frac{3x}{25\sqrt{1 - \frac{x^2}{25}}}\\
\left(2 - \frac{16x}{25}\right)^2 &= \frac{9x^2}{625 - 25x^2}\\
\left(2 - \frac{16x}{25}\right)^2 (625 - 25x^2) &= 9x^2\\
\left(2 - \frac{16x}{25}\right)^2 (625 - 25x^2) - 9x^2 &= 0\\
-\frac{256}{25}x^4 + 64x^3 + 147x^2 - 1600x + 2500 &= 0.
\end{aligned}$$

Using our favorite computer algebra system, we have that the only positive real solutions to this quartic equation is $x \approx 2.56478$, whence $y \approx 2.57524$.

Now that we know what point the normal line passes through, we get that the equation for this normal line (with everything rounded to two decimal places is

$$y - 1 = \frac{2.57524 - 1}{2.56478 - 2}(x-2) \tag{3}$$

$$\Rightarrow \quad y = 2.79x - 4.58 \tag{4}$$

b. Reflecting Equation 4 about the y-axis, the equation of the line passing through $(-p, q)$ is

$$y = -2.79x - 4.58. \tag{5}$$

Reflecting Equation 5 about the x-axis, the equation of the line passing through $(-p, -q)$ is

$$y = 2.79x + 4.58. \tag{6}$$

Reflecting Equation 6 about the y-axis, the equation of the line passing through $(p, -q)$ is

$$y = -2.79x + 4.58.$$