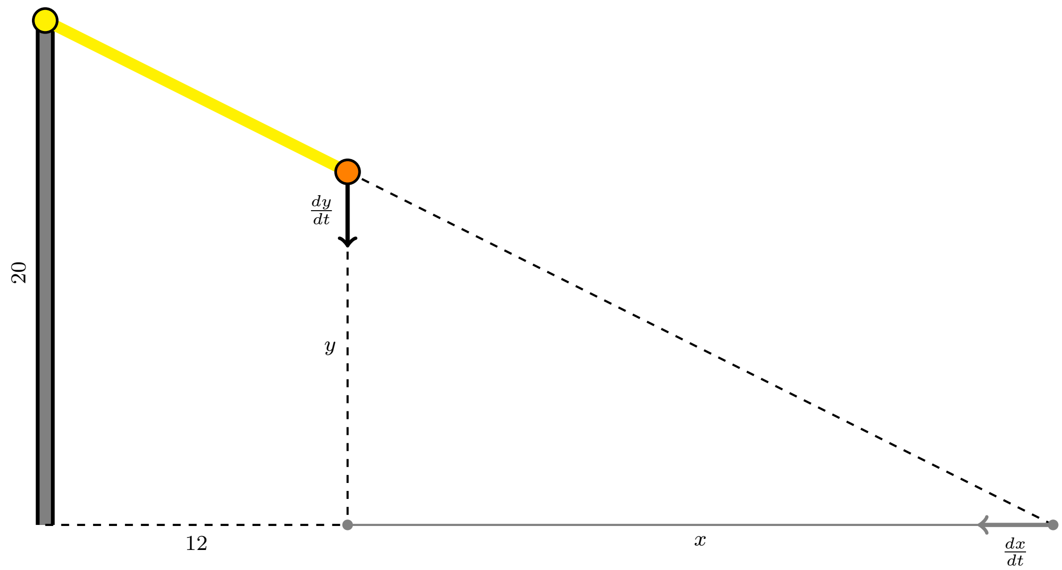
1.

a.



b. Let $x = x(t)$ be the length of the shadow at time t, and let $y = y(t)$ be the height of the ball at time t. The question asks about how $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are related, so we first need to relate x and y. From similar triangles, we have that

$$\begin{aligned} \frac{y}{x} &= \frac{20}{x+12} \\ \Rightarrow \quad y &= \frac{20x}{x+12}. \end{aligned}$$

Taking a derivative of both sides with respect to t and applying the quotient rule on the right-hand side, we have

$$\begin{aligned} \frac{d}{dt}[y] &= \frac{d}{dt}\left[\frac{20x}{x+12}\right] \\ \frac{dy}{dt} &= \frac{20(x+12) - 20x}{(x+12)^2} \cdot \frac{dx}{dt} \\ &= \frac{240}{(x+12)^2} \cdot \frac{dx}{dt} \\ \Rightarrow \quad \frac{dx}{dt} &= \frac{(x+12)^2}{240} \cdot \frac{dy}{dt}. \end{aligned} \tag{1}$$

Since the ball is dropped from rest and we know that gravity is $-9.8\,\mathrm{m/s^2}$, we use the standard kinematic equation for vertical position of an object dropped from height $h = 20\,\mathrm{m}$ (we've seen this equation before in examples):

$$y = \frac{1}{2}gt^2 + h = -4.9t^2 + 20,$$

which means that at time $t = 1\,\text{s}$, $y = 15.1\,\text{m}$. To find x at $t = 1$, we use this value of y with Equation **??** to get

$$\begin{aligned} 15.1 &= \frac{20x}{x+12} \\ 15.1(x+12) &= 20x \\ 181.2 &= 4.9x \\ \Rightarrow \quad x &= \frac{1812}{49}\,\text{m} \approx 36.98\,\text{m} \end{aligned}$$

Now, since the first derivative of position is velocity, the vertical velocity of the ball at time t is given by

$$\frac{dy}{dt} = \frac{d}{dt}\left[-4.9t^2 + 20\right] = -9.8t,$$

and at $t = 1\,\text{s}$, we have

$$\frac{dy}{dt} = -9.8.$$

We now have all of the information we need to solve for $\frac{dx}{dt}$. Using Equation 1,

$$\frac{dx}{dt} = \frac{\left(\frac{1812}{49} + 12\right)^2}{240}(-9.8)\,\text{m/s} \approx -97.959\,\text{m/s}.$$

The negative number here makes sense too – as the ball gets closer to the ground, the shadow is shrinking.