

MAT265 HOMEWORK 12 (SOLUTIONS)

1. Recall that $a(t) = s''(t)$. Taking two antiderivatives (in their most general forms), we get

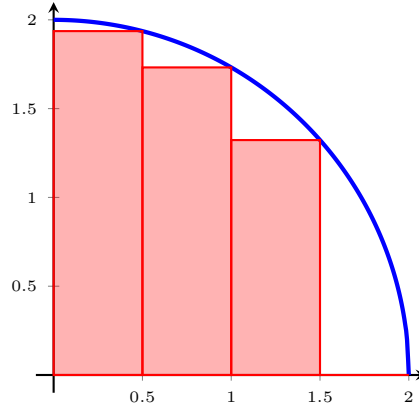
$$\begin{aligned} a(t) &= 12t^2 + 6t - 6 \\ v(t) &= 4t^3 + 3t^2 - 6t + C_1 \\ s(t) &= t^4 + t^3 - 3t^2 + C_1t + C_2. \end{aligned}$$

Since $s(0) = 4$, we see that $C_2 = 4$. And since $s(1) = 2$, we get that $C_1 = -1$. So our position equation is

$$s(t) = t^4 + t^3 - 3t^2 - t + 4.$$

2. a. We're partitioning the interval $[0, 2]$ into 4 rectangles, so $\Delta x = \frac{2-0}{4} = 0.5$. We have then that

$$x_0 = 0, \quad x_1 = 0.5, \quad x_2 = 1, \quad x_3 = 1.5, \quad x_4 = 2.$$



For a right endpoint estimate, we have

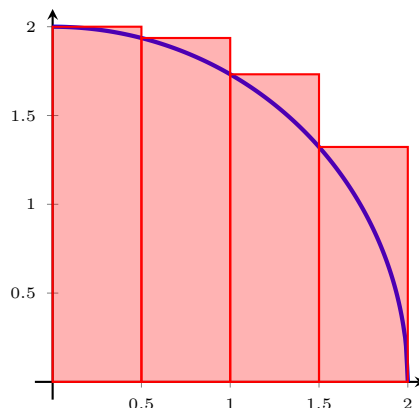
$$\begin{aligned} R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \\ &= f(x_1)(0.5) + f(x_2)(0.5) + f(x_3)(0.5) + f(x_4)(0.5) \\ &\approx (1.936)(0.5) + (1.732)(0.5) + (1.323)(0.5) + (0)(0.5) \\ &\approx 2.496 \end{aligned}$$

This is an underestimate.

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- b. We're partitioning the interval $[0, 2]$ into 4 rectangles, so $\Delta x = \frac{2-0}{4} = 0.5$. We have then that

$$x_0 = 0, \quad x_1 = 0.5, \quad x_2 = 1, \quad x_3 = 1.5, \quad x_4 = 2.$$



For a left endpoint estimate, we have

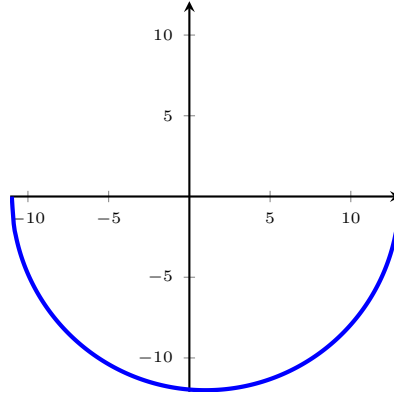
$$\begin{aligned} L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x \\ &= f(x_0)(0.5) + f(x_1)(0.5) + f(x_2)(0.5) + f(x_3)(0.5) \\ &\approx (2)(0.5) + (1.936)(0.5) + (1.732)(0.5) + (1.323)(0.5) \\ &\approx 3.496 \end{aligned}$$

This is an overestimate.

3. $\int_{-\pi/2}^{3\pi/2} \frac{\sin x}{x} dx$
 4. $\int_{-1}^1 \frac{x}{1+x^2} dx$

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5.



Setting $y = -\sqrt{144 - (x - 1)^2}$, we see that $y^2 = 144 - (x - 1)^2$, which rearranges to

$$(x - 1)^2 + y^2 = 144.$$

The function we're looking at is just the bottom half of the circle of radius 12 centered at $(1, 0)$ (hence a negative signed area). Thus

$$\int_{-11}^{13} -\sqrt{144 - (x - 1)^2} dx = -\frac{1}{2}\pi(12)^2 = -72\pi.$$