1. **a.** The graph of the derivative is positive for $(4, 6)$ and $(8, \infty)$, so f is increasing on both of these intervals.

b. Since $f'(x) = 0$ for $x = 4, 6, 8$ and the derivative switches sign at each of these, by the First Derivative Test, $x = 4$ and $x = 8$ are local minima; $x = 8$ is a local maximum.

c. The graph is increasing on $(-\infty, 2.1283)$, $(3.0312, 5.1342)$, and $(7.3063, \infty)$, so the function is concave upward on all of these intervals. The graph is decreasong on $(2.1283, 3.0312)$ and $(5.1342, 7.3063)$, so the function is concave downward on these intervals.

d. By the concavity test, using the information for part (c), we have that inflection points occur at $x = 2.1283$, $x = 3.0312$, $x = 5.1342$, and $x = 7.3063$.

2. **a.** Taking the first derivative, we have

$$f'(x) = 36 + 6x - 6x^2 = 6(3 - x)(2 + x)$$

Using our fa$f'(x) = 0$, we see that we have critical points at $x = -2$ and $x = 3$. Testing values in between these points, we see that $f'(x) < 0$ on $(-\infty, -2)$ and $(3, \infty)$; $f'(x) > 0$ on $(-2, 3)$. Thus f is decreasing on $(-\infty, -2)$ and $(3, \infty)$; f is increasing on $(-2, 3)$.

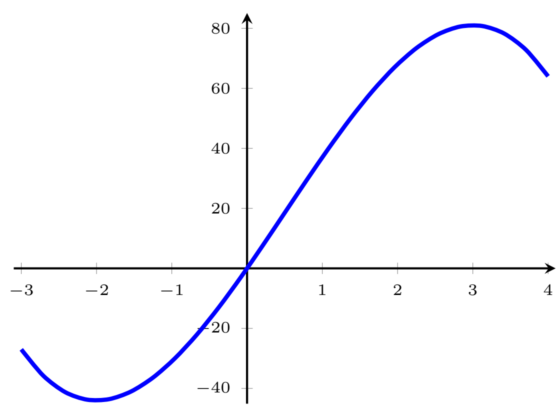
b. By the First Derivative Test and part (a), we have a local maximum at $x = 3$ with value 81 and a local minimum at $x = -3$ with value -44.

c. Taking another derivative, we have

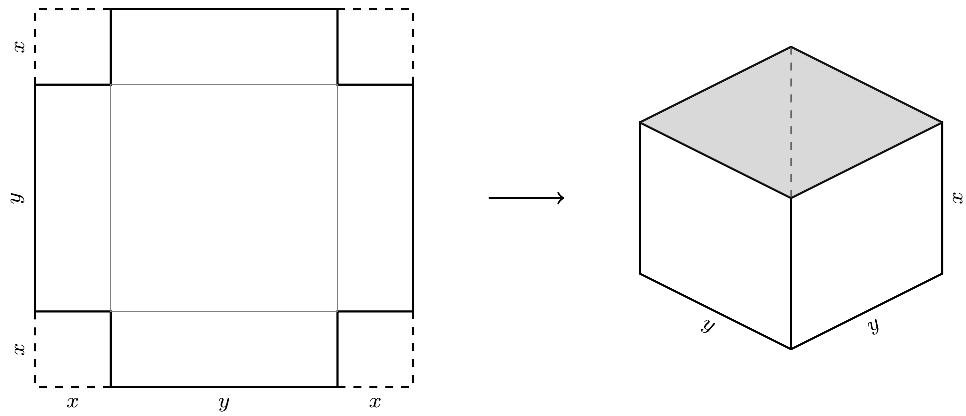
$$f''(x) = 6 - 12x = 6(1 - 2x),$$

so $f''(x) = 0$ at $x = \frac{1}{2}$. Testing values on either side of this point, we have that $f''(x) < 0$ on $\left(-\infty, \frac{1}{2}\right)$ and $f''(x) > 0$ on $\left(\frac{1}{2}, \infty\right)$. Thus f is concave downward on $\left(-\infty, \frac{1}{2}\right)$ and concave upward on $\left(\frac{1}{2}, \infty\right)$. This means that we have an inflection point at $x = \frac{1}{2}$.

d.



3. **a.**



b. The volume of the box is given by $V = xy^2$

c. Since the cardboard is $4\,\text{ft} \times 4\,\text{ft}$, we have $2x + y = 4$.

d. Rearranging to solve for y, we get $y = 4 - 2x$. Thus

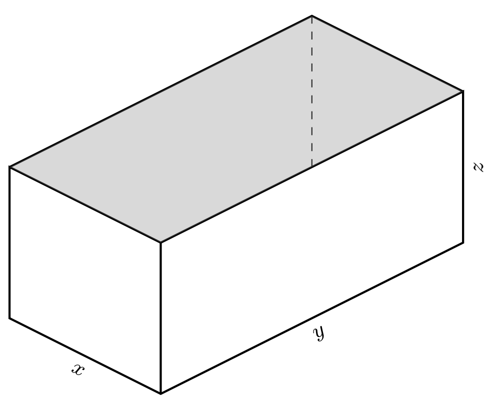
$$V = xy^2 = x(4 - 2x)^2 = 4x^3 - 16x^2 + 16x.$$

e. We see that $0 \le x \le 2$, but since it's easy to see that the box will have 0 volume if $x = 0$ or $x = 2$, then we're really looking to find a maximum value on $(0, 2)$, i.e., we're looking to find a local max. So

$$V'(x) = 12x^2 - 32x + 16 = 4(3x^2 - 8x + 4) = 4(3x - 2)(x - 2)$$

$V'(x) = 0$ on the interval $(0, 2)$ precisely when $x = \frac{2}{3}$, and this corresponds to a maximum with value $\frac{128}{27} \approx 4.7407\,\text{ft}^3$.

4.



We know that

$$y = 2x \quad \text{and}$$
$$xyz = 15 \Rightarrow z = \frac{15}{xy} \Rightarrow z = \frac{15}{2x^2}.$$

The base costs $14(xy)$, two of the sides cost $7(xz)$ each, and the other two sides cost $7(yz)$ each. Altogether, the cost is

$$C = 14xy + 2 \cdot 7xz + 2 \cdot 7yz = 14x(2x) + 14x\left(\frac{15}{2x^2}\right) + 14(2x)\left(\frac{15}{2x^2}\right) = \frac{28x^3 + 315}{x}.$$

We're now looking to minimize this function on the interval $(0, \infty)$. Taking the derivative, we get

$$C'(x) = \frac{56x^3 - 315}{x^2},$$

which has a critical point at $x = \sqrt[3]{\frac{315}{56}} \approx 1.7784$. This point is a minimum, and the corresponding value is \$265.68.

5. Let (x, y) be a point on $y = x^4$. The distance from $(0, 7)$ to (x, y) is given by

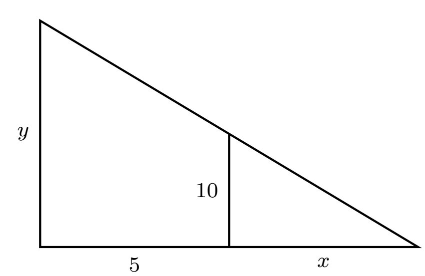
$$z = \sqrt{(x-0)^2 + (7-y)^2} = \sqrt{x^2 + (7-x^4)^2}.$$

To minimize, we take the derivative to get

$$z'(x) = \frac{2x + 2(7-x^4)(4x^3)}{\sqrt{x^2 + (7-x^4)^2}}$$

With our favorite computer algebra system, we see that we have critical points at $x = 0$, $x \approx \pm 0.189$, and $x \approx \pm 1.621$. However, only $x \approx \pm 1.621$ correspond to local minima. Thus the points $(\pm 1.621, 6.904)$ are the closest points on $y = x^4$

6. Let x be the distance from the wall to the base of the ladder, and y the height from the ground to the top of the ladder resting on the wall (as in the picture below).



By similar triangles, we have

$$\frac{y}{x+5} = \frac{10}{x} \quad \Rightarrow \quad y = \frac{10x + 50}{x}.$$

With the Pythagorean Theorem, the length of the ladder is

$$z = \sqrt{(5+x)^2 + y^2} = \sqrt{(5+x)^2 + \left(\frac{10x+50}{x}\right)^2}.$$

Certainly we need $x > 0$, so to minimize the length of the ladder on the interval $(0, \infty)$, we take the derivative to get

$$z'(x) = \frac{2(5+x) + 2\left(\frac{10x+50}{x}\right)\left(-\frac{50}{x^2}\right)}{\sqrt{(5+x)^2 + \left(\frac{10x+50}{x}\right)^2}} = \frac{(x+5)(x^3-500)}{x^3\sqrt{(5+x)^2 + \left(\frac{10x+50}{x}\right)^2}}.$$

The only critical point for z in our interval is thus $x = \sqrt[3]{500} \approx 7.937$, which corresponds to a minimum with value $20.810\,\text{ft}$.