

MAT265 HOMEWORK 09 (SOLUTIONS)

1. Recall that $\frac{d}{dx}|x| = \frac{x}{|x|}$. From the chain rule we have that

$$g'(t) = \frac{16t - 12}{|3 - 4t|}.$$

This derivative is always nonzero and is undefined for $t = \frac{3}{4}$. So, our only critical point is $t = \frac{3}{4}$.

2. By the power rule,

$$h'(x) = -\frac{1}{3}x^{-4/3} + \frac{2}{3}x^{-5/3} = \frac{-x^{1/3} + 2}{3x^{5/3}}.$$

We see that $h'(x) = 0$ when $x = 8$ and h' is undefined for $x = 0$. However, 0 is not in the domain of h , so the only critical point is as 8.

3. By the product rule, we have that

$$f'(x) = -5x^{-6} \ln x + \frac{x^{-5}}{x} = \frac{-5 \ln x}{x^6} + \frac{1}{x^6} = \frac{-5 \ln x + 1}{x^6}.$$

We see that $f'(x) = 0$ for $x = e^{1/5}$ and f' is undefined at $x = 0$. However, 0 is not in the domain of f , so the only critical point is as $x = e^{1/5}$.

4. Taking the first derivative, we have

$$f'(t) = 1 - \frac{1}{2} \csc^2\left(\frac{1}{2}t\right)$$

We have that $\sin\left(\frac{1}{2}t\right) \neq 0$ on $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$, so $f'(t)$ is continuous on this closed interval. We also have that $f'\left(\frac{\pi}{2}\right) = f'\left(\frac{3\pi}{2}\right) = 0$, so f has two critical points on this interval. Testing these critical points and the endpoints,

$$\begin{aligned} f\left(\frac{\pi}{4}\right) &\approx 3.1996, \\ f\left(\frac{\pi}{2}\right) &\approx 2.5708, \\ f\left(\frac{3\pi}{2}\right) &\approx 3.7124, \\ f\left(\frac{7\pi}{4}\right) &\approx 3.0836, \end{aligned}$$

by the Extreme Value Theorem, we have that the minimum value 2.5708 occurs at $t = \frac{\pi}{2}$, and the maximum value 3.7124 occurs at $t = \frac{3\pi}{2}$.

5. Taking the first derivative, we have

$$f'(x) = \frac{1}{x} - 1.$$

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Since $x \neq 0$ on $[\frac{1}{2}, 2]$, we have that $f'(x)$ is continuous on this interval. As $f'(1) = 0$, there is a single critical point on this interval. Testing this critical point and the endpoints,

$$f\left(\frac{1}{2}\right) \approx -1.1931,$$

$$f(1) = -1,$$

$$f(2) \approx -1.3069,$$

by the Extreme Value Theorem, we have that the minimum value -1.3069 occurs at $x = 2$, and the maximum value -1 occurs at $x = 1$.

6. Taking the first derivative, we have

$$f'(s) = 1 + \frac{2}{1 + s^2}.$$

Since $s^2 \neq -1$ on \mathbb{R} , we have that $f'(s)$ is continuous on the interval $[0, 4]$. There are no critical points on this interval. Testing the endpoints,

$$f(0) = 0,$$

$$f(4) \approx 6.6516,$$

by the Extreme Value Theorem, we have that the minimum value 0 occurs at $s = 0$, and the maximum value 6.6516 occurs at $s = 4$.