1. Recall that the area of a disk of radius r is given by $A = \pi r^2$. Using differentials, we see that $dA = 2\pi r\,dr$.

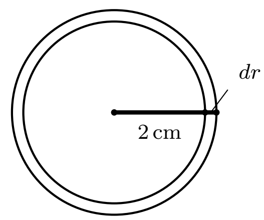
a. When $r = 40\,\text{cm}$ and $dr = 0.3\,\text{cm}$, we get that the maximum propogated error in measurement of the area is

$$dA = 2\pi r\,dr = 2\pi(40)(0.3) \approx 75.398\,\text{cm}^2.$$

b. The relative error (resp. percentage error) in measurement of the Area is given by

$$\frac{dA}{A} = \frac{2\pi r\,dr}{\pi r^2} = \frac{2dr}{r} = \frac{0.6}{40} = 0.015\ (= 1.5\%).$$

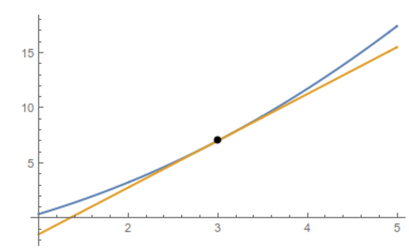
2.



Recall that the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. Using differentials, we see that $dV = 4\pi r^2\,dr$. When $r = \frac{4\,\text{cm}}{2} = 2\,\text{cm}$ and $dr = 2.5\,\text{mm} = 0.25\,\text{cm}$, we estimate that the amount of chocolate needed is

$$dV = 4\pi(2)^2(0.25) = 4\pi\,\text{cm}^3 \approx 12.566.$$

3.



Shown above: in blue, the graph of $y = g(x)$; in gold, the tangent line at $(3, g(3))$.

a. The linear approximation at $x = 3$ is

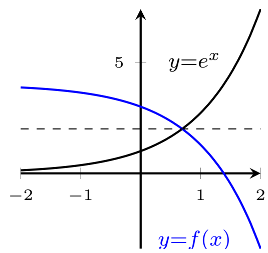
$$L(x) = g'(4)(x-3) + g(3) = \sqrt{x^2+2x+3}(x-3) + 7.$$

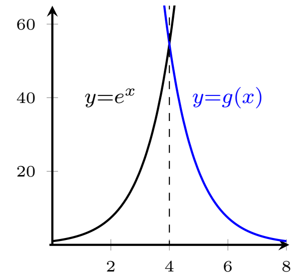
Using this, we get

$$g(2.9) \approx L(2.9) = 6.582 \quad \text{and} \quad g(3.05) \approx L(3.05) = 7.214.$$

b. Since $g'(x)$ is always positive and it is increasing, it must be that the tangent line is always below the graph of $y = g(x)$. This means our estimates are always too small.

4.





a. Multiplying the function by -1 flips about the line $y = 0$, and then adding 4 to the function translates the graph up 4 units. So we get the net effect of reflecting about the line $y = 2$.

$$f(x) = -e^x + 4.$$

b. Subtracting 8 from the function's argument translates the graph right 8 units, and then multiplying the function's argument by -1 flips about the line $x = 0$. So we get the net effect of reflecting about the line $x = 4$.

$$g(x) = e^{-(x-8)}.$$

5. Since $\sin(x)$ has domain $(-\infty, \infty)$ the domain of $\sin(e^{-x})$ is equal to the domain of e^{-x}, which is $(-\infty, \infty)$.

6. Let $t = -x^2$. Note that as $x \to \infty$, we have $t \to -\infty$. Thus, since $e > 1$,

$$\lim_{x\to\infty} e^{-x^2} = \lim_{t\to-\infty} e^t = 0.$$

7.

$$\begin{aligned}
\lim_{x\to\infty} \frac{2 + 10^x}{3 - 10^x} &= \lim_{x\to\infty} \frac{10^x\left(2 \cdot \frac{1}{10^x} + 1\right)}{10^x\left(3 \cdot \frac{1}{10^x} - 1\right)} \\
&= \lim_{x\to\infty} \frac{2 \cdot \frac{1}{10^x} + 1}{3 \cdot \frac{1}{10^x} - 1} \\
&= \frac{2 \cdot \left[\lim_{x\to\infty} \frac{1}{10^x}\right] + 1}{3 \cdot \left[\lim_{x\to\infty} \frac{1}{10^x}\right] - 1} \\
&= \frac{1}{-1} = -1.
\end{aligned}$$