1. (a) Since $g(x) = f(x)\sin(x)$ is a product of two functions, we calculate g' using the product rule:

$$\begin{aligned} g'(x) &= \frac{d}{dx}[f(x)\sin(x)] \\ &= \frac{d}{dx}[f(x)] \cdot \sin(x) + f(x) \cdot \frac{d}{dx}[\sin(x)] \\ &= f'(x)\sin(x) + f(x)\cos(x). \end{aligned}$$

Now we can evaluate the derivative at $x = \frac{\pi}{4}$:

$$\begin{aligned} g'\left(\frac{\pi}{4}\right) &= f'\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) \\ &= (-3)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{-5\sqrt{2}}{4}. \end{aligned}$$

(b) Since $h(x) = \frac{\cos(x)}{f(x)}$ is a quotient of two functions, we calculate h' using the quotient rule:

$$\begin{aligned} h'(x) &= \frac{d}{dx}\left[\frac{\cos(x)}{f(x)}\right] \\ &= \frac{\frac{d}{dx}[\cos(x)]f(x) - \cos(x)\frac{d}{dx}[f(x)]}{[f(x)]^2} \\ &= \frac{f(x)[-\sin(x)] - f'(x)\cos(x)}{[f(x)]^2} \\ &= -\frac{f(x)\sin(x) + f'(x)\cos(x)}{[f(x)]^2}. \end{aligned}$$

Now we can evaluate the derivatives at $x = \frac{\pi}{4}$:

$$\begin{aligned} h'\left(\frac{\pi}{4}\right) &= -\frac{f\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)}{\left[f\left(\frac{\pi}{4}\right)\right]^2} \\ &= \frac{\frac{5\sqrt{2}}{4}}{(-3)^2} \\ &= \frac{5\sqrt{2}}{36} \end{aligned}$$

2. The following numbers can be found using the appropriate derivative rules and the given values.

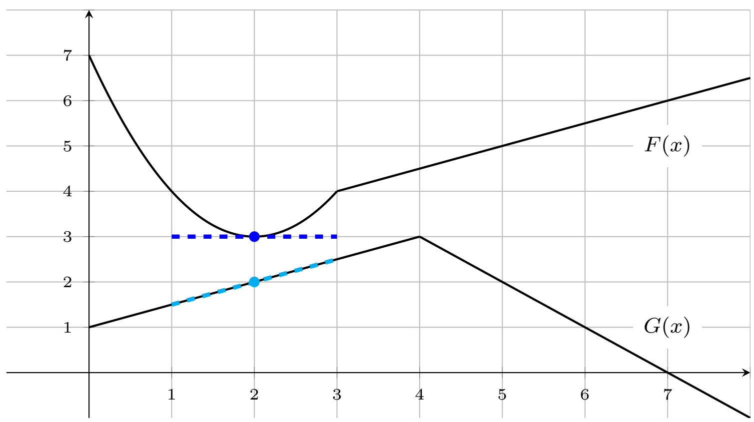
(a) Sum rule: $(f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$

(b) Product rule: $(fg)'(3) = f(3)g'(3) + g(3)f'(3) = (4)(5) + (2)(-6) = 8$

(c) Quotient rule: $(\frac{f}{g})'(3) = \dfrac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2} = \dfrac{(2)(-6) - (4)(5)}{4} = \dfrac{-32}{4} = -8$

(d) Quotient rule: $(\frac{g}{f})'(3) = \dfrac{f(3)g'(3) - g(3)f'(3)}{[f(3)]^2} = \dfrac{(4)(5) - (-6)(2)}{16} = \dfrac{32}{16} = 2$

3. The following numbers can be found using the appropriate derivative rules and information from the graph. We need to estimate the values for $F'(2)$ and $G'(2)$:



(a) From the graph we see that $F(2) = 3$ and $G(2) = 2$. Recalling that $F'(2)$ is the slope of the tangent line of the graph of $F(x)$ at $x = 2$, from the graph we estimate that $F'(2) = 0$ and that $G'(2) = \frac{1}{2}$. Thus,

$$P'(2) = F'(2)G(2) + G'(2)F(2) = (0)(2) + \left(\frac{1}{2}\right)(3) = \frac{3}{2}.$$

(b) Since $G(7) = 0$, $Q'(7)$ does not exist (otherwise we would be dividing by 0).

4. Let g be a differentiable function then,

(a) $y' = 3x^2 g(x) + x^3 g'(x)$

(b) $y' = \dfrac{x^4 g'(x) - 4x^3 g(x)}{x^8}$

(c) $y' = \dfrac{g(x)(2x) - x^2 g'(x)}{[g(x)]^2}$

(d) $y' = \dfrac{\sqrt{x}(g(x) + xg'(x)) - (1 + xg(x))\frac{1}{2\sqrt{x}}}{x}$

5. We notice that h is a composite function. From the chain rule it follows that

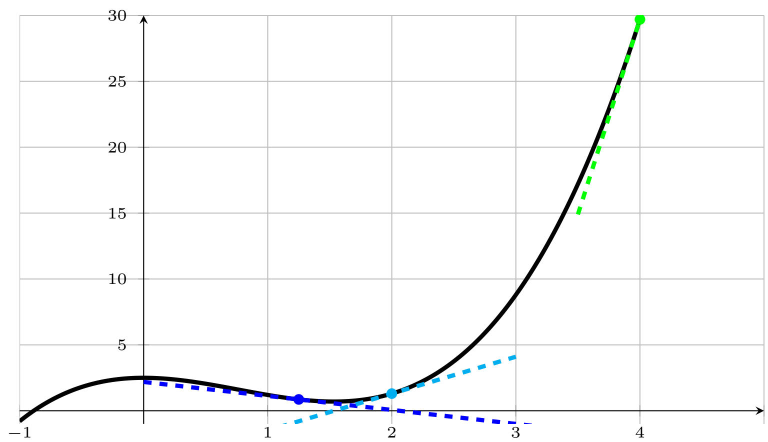
$$h'(x) = \frac{f'(x)}{\sqrt{3 + 2f(x)}},$$

and in particular,

$$h'(2) = \frac{5}{3}.$$

6. (a) $F'(2) = f'(f(2))\, f'(2) = f'(1)(5) = (4)(5) = 20$
(b) $G'(3) = g'(g(3))\, g'(3) = g'(2)(9) = (7)(9) = 63$
(c) $H'(2) = g'(f(2))\, f'(2) = g'(1)(5) = (6)(5) = 30$

7. Using the chain rule, it follows that $h'(x) = f'(f(x))\, f'(x)$ and $g'(x) = f'(x^2)\,(2x)$. Therefore $h'(2) = f'\left(\frac{5}{4}\right) f'(2)$ and $g'(2) = 4f'(4)$. We use the graph to estimate these values:



From the graph, it appears that $f'\left(\frac{5}{4}\right) = -1$, $f'(2) = 3$, $f'(4) = 30$. Thus, $h'(2) = (-1)(3) = -3$ and $g'(2) = 4(30) = 120$.

8. Let f be differentiable on $\mathbb{R}$ and α be a real number. Then
(a) $F'(x) = f'(x^\alpha)(\alpha x^{\alpha-1})$
(b) $G'(x) = \alpha[f(x)]^{\alpha-1}\, f'(x)$

9. Let g be a twice differentiable function and $f(x) = g(x^3)\sin(x)$. Then

$$f'(x) = g\left(x^3\right)\cos(x) + g'\left(x^3\right)\left(3x^2\right)\sin(x)$$

and

$$f''(x) = -g\left(x^3\right)\sin(x) + g'\left(x^3\right)\left(3x^2\right)\cos(x) + 6x[g'(x^3)\sin(x)] \\ + \left(3x^2\right)\left[g''\left(x^3\right)\left(3x^2\right)\sin(x) + g'\left(x^3\right)\cos(x)\right].$$