MAT265 Homework 03 (Solutions)

1. Since $x^{5/2} = \sqrt{x^5}$, we have that the domain for $f(x)$ is $[0, \infty)$. The derivative is given by

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{(x+h)^{5/2} - x^{5/2}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{(x+h)^5} - \sqrt{x^5}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{(x+h)^5} - \sqrt{x^5}}{h} \left(\frac{\sqrt{(x+h)^5} + \sqrt{x^5}}{\sqrt{(x+h)^5} + \sqrt{x^5}} \right) \\ &= \lim_{h\to 0} \frac{(x+h)^5 - x^5}{h\left(\sqrt{(x+h)^5} + \sqrt{x^5}\right)} \\ &= \lim_{h\to 0} \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h\left(\sqrt{(x+h)^5} + \sqrt{x^5}\right)} \\ &= \lim_{h\to 0} \frac{5x^4 + 10x^3h^1 + 10x^2h^2 + 5xh^3 + h^4}{\left(\sqrt{(x+h)^5} + \sqrt{x^5}\right)} \\ &= \frac{5x^4 + 0 + 0 + 0 + 0}{\sqrt{(x+0)^5} + \sqrt{x^5}} = \frac{5x^4}{2\sqrt{x^5}} = \frac{5}{2}x^{3/2}. \end{aligned}$$

Again, since $x^{3/2} = \sqrt{x^3}$, but $f'(0)$ is not defined, we have that the domain for $f'(x)$ is $(0, \infty)$.

2. f is **a.**

f' is **d.** In (a), the slopes of the tangent lines at $x = 2$ and $x = -2$ are both 0.

f'' is **b.** In (c), the slope of the tangent line at $x = 2$ is positive and the slope of the tangent line at $x = -2$ is negative.

f''' is **c.** In (b), the slopes of the tangent lines at $x = 2$ and $x = -2$ are both positive.

3. A horizontal line has slope 0, so we're looking for places where $y'(x) = 0$. First we calculate that $y'(x) = 6x^2 + 2x - 4$. Then

$$0 = 6x^2 + 2x - 4 = (2x + 2)(3x - 2)$$

and thus $y'(x) = 0$ when $x = -1$ and $x = \frac{2}{3}$.

4. To involve the slope, we calculate the derivative $y'(x) = 2ax + b$. We're given that

$$\begin{aligned} y'(2) &= 4 = 4a + b \\ y'(-1) &= -6 = -2a + b, \end{aligned}$$

so solving this system, we get that $a = \frac{5}{3}$ and $b = -\frac{8}{3}$. We're also given that

$$y(5) = 1 = 25a + 5b + c = \frac{125}{3} - \frac{40}{3} + c,$$

so solving for c, we get that $c = -\frac{82}{3}$. Thus the equation of the parabola satisfying these conditions is

$$y = \frac{5}{3}x^2 - \frac{8}{3}x - \frac{82}{3}.$$