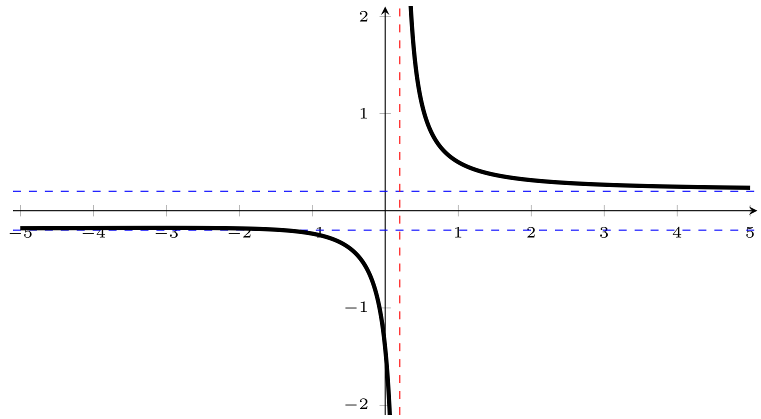
1. The graph of the function is below.

a.



From the above graph, it looks like we have two horizontal asymptotes and one vertical asymptote. As such, we estimate the following:

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+x+2}}{5x-1}\approx -0.2 \quad \text{and} \quad \lim_{x\to\infty}\frac{\sqrt{x^2+x+2}}{5x-1}\approx 0.2.$$

b. The tables below appear to confirm our estimates from part (a).

x	$f(x)$
-1	-0.235702
-10	-0.188072
-100	-0.19862
-1000	-0.19986
-10,000	-0.199986

x	$f(x)$
1	0.5
10	0.21598
100	0.20142
1000	0.20014
10000	0.200014

c. The key here is to recall that $\sqrt{x^2}=|x|$, and then we'll appeal to the piecewise definition of the absolute value of x.

$$\begin{aligned}
\lim_{x\to-\infty}\frac{\sqrt{x^2+x+2}}{5x-1} &= \lim_{x\to-\infty}\frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{2}{x^2}\right)}}{x\left(5-\frac{1}{x}\right)}\\
&= \lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)}\\
&= \lim_{x\to-\infty}\frac{|x|\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)}\\
&= \lim_{x\to-\infty}\frac{-x\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)} \qquad \text{(since } |x|=-x \text{ for } x<0)\\
&= \lim_{x\to-\infty}-\frac{\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{5-\frac{1}{x}}
\end{aligned}$$

$$= -\frac{\sqrt{1+0+0}}{5-0} = -\frac{1}{5}.$$

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt{x^2+x+2}}{5x-1} &= \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{2}{x^2}\right)}}{x\left(5-\frac{1}{x}\right)} \\
&= \lim_{x\to\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)} \\
&= \lim_{x\to\infty} \frac{|x|\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)} \\
&= \lim_{x\to\infty} \frac{x\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{x\left(5-\frac{1}{x}\right)} \qquad \text{(since } |x| = x \text{ for } x \geq 0\text{)} \\
&= \lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}}{5-\frac{1}{x}} \\
&= \frac{\sqrt{1+0+0}}{5-0} = \frac{1}{5}.
\end{aligned}$$

2. To find the horizontal asymptotes, we take the limits as $x \to -\infty$ and $x \to \infty$. Again, we'll need to recall that $\sqrt{x^2} = |x|$ and appeal to the piecewise definition of the absolute value.

$$\begin{aligned}
\lim_{x\to-\infty} \frac{x-5}{\sqrt{x^2-x+4}} &= \lim_{x\to-\infty} \frac{x\left(1-\frac{5}{x}\right)}{\sqrt{x^2\left(1-\frac{1}{x}+\frac{4}{x^2}\right)}} \\
&= \lim_{x\to-\infty} \frac{x\left(1-\frac{5}{x}\right)}{\sqrt{x^2}\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= \lim_{x\to-\infty} \frac{x\left(1-\frac{5}{x}\right)}{|x|\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= \lim_{x\to-\infty} \frac{x\left(1-\frac{5}{x}\right)}{-x\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \qquad \text{(since } |x| = -x \text{ for } x < 0\text{)} \\
&= \lim_{x\to-\infty} -\frac{1-\frac{5}{x}}{\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= -\frac{1-0}{\sqrt{1-0+0}} = -1,
\end{aligned}$$

and

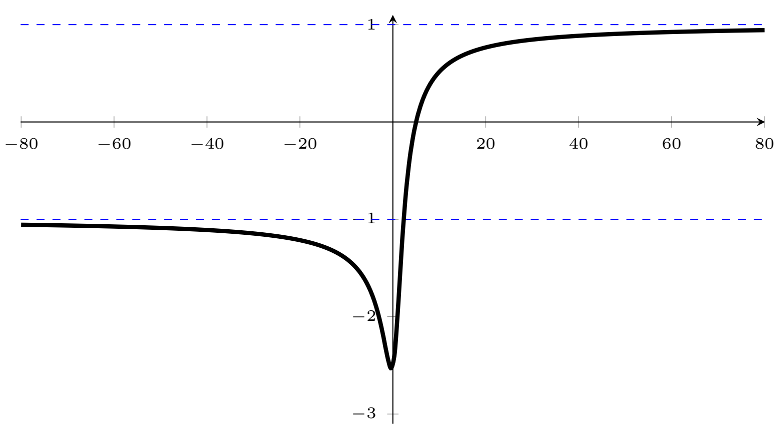
$$\lim_{x\to\infty} \frac{x-5}{\sqrt{x^2-x+4}} = \lim_{x\to\infty} \frac{x\left(1-\frac{5}{x}\right)}{\sqrt{x^2\left(1-\frac{1}{x}+\frac{4}{x^2}\right)}}$$

$$
\begin{aligned}
&= \lim_{x\to\infty} \frac{x\left(1-\frac{5}{x}\right)}{\sqrt{x^2}\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= \lim_{x\to\infty} \frac{x\left(1-\frac{5}{x}\right)}{|x|\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= \lim_{x\to\infty} \frac{x\left(1-\frac{5}{x}\right)}{x\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \qquad \text{(since } |x| = x \text{ for } x \geq 0\text{)} \\
&= \lim_{x\to\infty} \frac{1-\frac{5}{x}}{\sqrt{1-\frac{1}{x}+\frac{4}{x^2}}} \\
&= \frac{1-0}{\sqrt{1-0+0}} = 1.
\end{aligned}
$$

So, we have two horizontal asymptotes: $y = -1$ and $y = 1$.

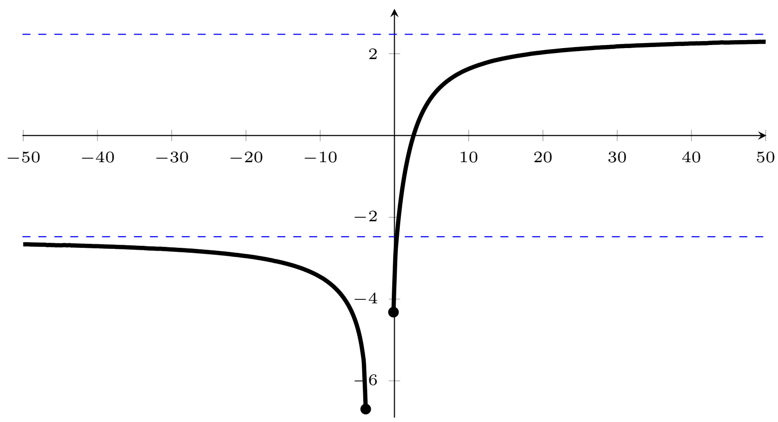
To check for the vertical asymptotes, we start by looking for x-values where the denominator is 0. Since the discriminant of $x^2 - x + 4$ is negative, this tells us that $x^2 - x + 4 > 0$ for all x, so there are no vertical asymptotes.

The graph of the function is below.



3.

a. With some work, we see that the function has domain $\left(-\infty, \frac{-\sqrt{14}-4}{2}\right] \cup \left[\frac{\sqrt{14}-4}{2}, \infty\right)$.



From the graph above, it looks like

$$\lim_{x\to\infty}\left(\sqrt{2x^2+8x+1}-\sqrt{2x^2+x+19}\right)\approx 2.5.$$

b. For the tables of values below,

x	$f(x)$
10	1.63031
100	2.38406
1000	2.46573
10,000	2.47396
100,000	2.47478

we estimate the following:

$$\lim_{x\to\infty}\left(\sqrt{2x^2+8x+1}-\sqrt{2x^2+x+19}\right)\approx 2.4750.$$

c. To solve the limits explicitly, we will use the conjugates of the square roots and again appeal to the fact that $\sqrt{x^2}=|x|$.

$$\begin{aligned}
&\lim_{x\to\infty}\left(\sqrt{2x^2+8x+1}-\sqrt{2x^2+x+19}\right)\\
&=\lim_{x\to\infty}\left(\sqrt{2x^2+8x+1}-\sqrt{2x^2+x+19}\right)\left(\frac{\sqrt{2x^2+8x+1}+\sqrt{2x^2+x+19}}{\sqrt{2x^2+8x+1}+\sqrt{2x^2+x+19}}\right)\\
&=\lim_{x\to\infty}\frac{(2x^2+8x+1)-(2x^2+x+19)}{\sqrt{2x^2+8x+1}+\sqrt{2x^2+x+19}}\\
&=\lim_{x\to\infty}\frac{7x-18}{\sqrt{x^2\left(2+\frac{8}{x}+\frac{1}{x^2}\right)}+\sqrt{x^2\left(2+\frac{1}{x}+\frac{19}{x^2}\right)}}\\
&=\lim_{x\to\infty}\frac{x\left(7-\frac{18}{x}\right)}{\sqrt{x^2}\left(\sqrt{2+\frac{8}{x}+\frac{1}{x^2}}+\sqrt{2+\frac{1}{x}+\frac{19}{x^2}}\right)}
\end{aligned}$$

$$= \lim_{x\to\infty} \frac{x\left(7-\frac{18}{x}\right)}{|x|\left(\sqrt{2+\frac{8}{x}+\frac{1}{x^2}}+\sqrt{2+\frac{1}{x}+\frac{19}{x^2}}\right)}$$

$$= \lim_{x\to\infty} \frac{x\left(7-\frac{18}{x}\right)}{x\left(\sqrt{2+\frac{8}{x}+\frac{1}{x^2}}+\sqrt{2+\frac{1}{x}+\frac{19}{x^2}}\right)} \qquad (\text{since } |x| = x \text{ when } x \geq 0)$$

$$= \lim_{x\to\infty} \frac{7-\frac{18}{x}}{\sqrt{2+\frac{8}{x}+\frac{1}{x^2}}+\sqrt{2+\frac{1}{x}+\frac{19}{x^2}}}$$

$$= \frac{7-0}{\sqrt{2+0+0}+\sqrt{2+0+0}} = \frac{7}{2\sqrt{2}} \approx 2.4749.$$

4. The average velocity over the interval $[a, b]$ is given by $\dfrac{s(b)-s(a)}{b-a}$.

a. **i.** $\dfrac{s(4)-s(3)}{4-3} = \dfrac{5-5.5}{4-3} = -0.5\,\text{m/s}$

ii. $\dfrac{s(4)-s(3.5)}{4-3.5} = \dfrac{5-5.125}{4-3.5} = -0.25\,\text{m/s}$

iii. $\dfrac{s(5)-s(4)}{5-4} = \dfrac{5.5-5}{5-4} = 0.5\,\text{m/s}$

iv. $\dfrac{s(4.5)-s(4)}{4.5-4} = \dfrac{5.125-5}{4.5-4} = 0.25\,\text{m/s}$

b. To find the instantaneous velocity at $t = 4$,

$$\begin{aligned} s'(4) = \lim_{t\to 4} \frac{s(t)-s(4)}{t-4} &= \lim_{t\to 4} \frac{\left(\frac{1}{2}t^2-4t+13\right)-5}{t-4} \\ &= \lim_{t\to 4} \frac{\frac{1}{2}t^2-4t+8}{t-4} \\ &= \lim_{t\to 4} \frac{\frac{1}{2}(t-4)^2}{t-4} \\ &= \lim_{t\to 4} \frac{1}{2}(t-4) = 0\,\text{m/s}. \end{aligned}$$

5. Recall that $g'(3)$ is the slope of the tangent line at the point $(3, g(3))$. Thus

$$\begin{aligned} y - g(3) &= g'(3)\,(x-3) \\ y + 5 &= 3(x-3) \\ y &= 3x - 14 \end{aligned}$$

is the equation of the tangent line to the curve $y = g(x)$ at the point $(3, -5)$.

6. Using the definition

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h},$$

it's not hard to see that $f(x) = \sqrt[3]{x}$ and $a = 27$.

7. Using the definition

$$f'(a) = \lim_{t\to a} \frac{f(t) - f(a)}{t - a},$$

it's not hard to see that $f(t) = t^3 - t^2$ and $a = 2$.

8. **a.** **i.** $\dfrac{19767 - 17003}{2013 - 2011} = 1382\,\text{stores/year}$

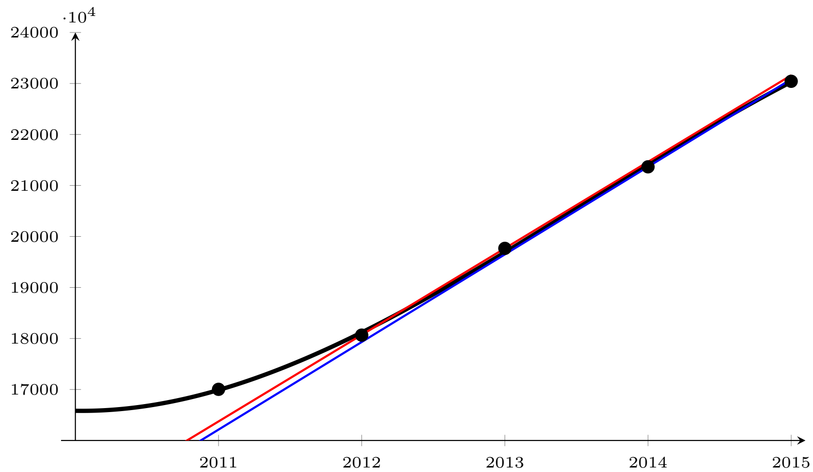
ii. $\dfrac{23043 - 21366}{2015 - 2014} = 1677\,\text{stores/year}$

iii. $\dfrac{21366 - 19767}{2014 - 2013} = 1599\,\text{stores/year}$

b. We'll take the average of the rates found in (ii) and (iii) above.

$$\frac{1599 + 1677}{2} = 1638\,\text{stores/year}.$$

c. First we graph the data given and try to come up with a reasonable curve passing through the 5 points.



Looking at the graph as we've drawn it, we estimate that the slope of the tangent line (in blue) at $t = 2014$ has a slope of about 1717 stores/year.

d. Looking at the graph in part (c), we estimate that the slope of the tangent line (in red) at $t = 2013$ has a slope of about 1696 stores/year. From this information, we estimate that the instantaneous rate of change is increasing by about 21 (stores/year)/year.