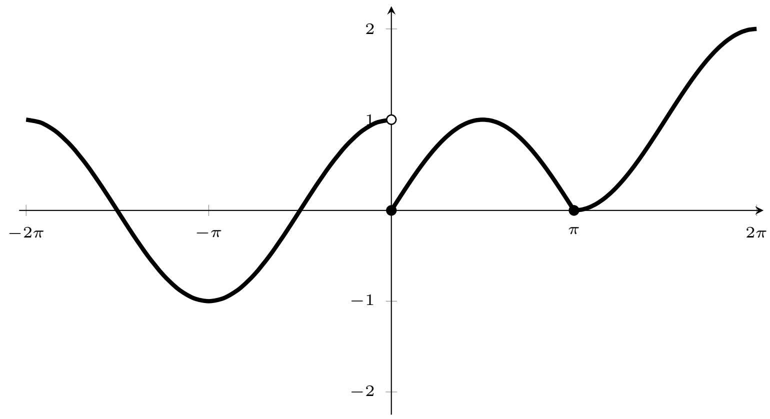
MAT265 Homework 01 (Solutions)

1. The graph of the function is below.



From the above graph we see that $\lim_{x\to a} f(x)$ exists for all real numbers a except when $a = 0$ as $\lim_{x\to 0^-} f(x) = 1$ and $\lim_{x\to 0^+} f(x) = 0$.

2. From the tables below, it looks like $\lim_{x\to 0} f(x) \approx 1.299$.

x	$f(x)$
-1	0.242424
-0.1	1.09165
-0.01	1.27677
-0.001	1.29701
-0.0001	1.29906

x	$f(x)$
1	8
0.1	1.54858
0.01	1.32221
0.001	1.30156
0.0001	1.29951

3.

a. What's wrong is that the two functions are not equal. In particular, the functions do not have the same domain. The left-hand side has domain $(-\infty, 3) \cup (3, \infty)$ and the right-hand side has domain $(-\infty, \infty)$.

b. Functions do not need to agree at a point to have their limits agree. Indeed, that is what is happening here.

4. The key here is combine the terms into a single fraction.

$$\begin{aligned}
\lim_{t\to 0}\left(\frac{1}{2t^2} - \frac{1}{2t^2+t^4}\right) &= \lim_{t\to 0}\left(\frac{2t^2+t^4-2t^2}{2t^2(2t^2+t^4)}\right) \\
&= \lim_{t\to 0}\left(\frac{t^4}{4t^4+2t^6}\right) \\
&= \lim_{t\to 0}\left(\frac{1}{4+2t^2}\right) \\
&= \frac{1}{4+2(0)^2} = \frac{1}{4}
\end{aligned}$$

5. The key here is to multiply the numerator and denominator by the conjugate of $\sqrt{x^2+144}-13$.

$$\begin{aligned}
\lim_{x\to-5}\frac{\sqrt{x^2+144}-13}{x+5} &= \lim_{x\to-5}\frac{\sqrt{x^2+144}-13}{x+5}\left(\frac{\sqrt{x^2+144}+13}{\sqrt{x^2+144}+13}\right)\\
&= \lim_{x\to-5}\frac{x^2+144-169}{(x+5)(\sqrt{x^2+144}+13)}\\
&= \lim_{x\to-5}\frac{x^2-25}{(x+5)(\sqrt{x^2+144}+13)}\\
&= \lim_{x\to-5}\frac{(x+5)(x-5)}{(x+5)(\sqrt{x^2+144}+13)}\\
&= \lim_{x\to-5}\frac{x-5}{\sqrt{x^2+144}+13}\\
&= \frac{(-5)-5}{\sqrt{(-5)^2+144}+13}\\
&= -\frac{5}{13}
\end{aligned}$$

6. Recall the definition of the absolute value tells us

$$|x-5| = \begin{cases} x-5 & \text{if } x-5\geq 0\\ -(x-5) & \text{if } x-5<0 \end{cases}.$$

So, we check the limits from the left and right.

$$\lim_{x\to5^-}\frac{3x-15}{|x-5|} = \lim_{x\to5^-}\frac{3x-15}{-(x-5)} = \lim_{x\to5^-}\frac{3(x-5)}{-(x-5)} = \lim_{x\to5^-} -3,$$

and

$$\lim_{x\to5^+}\frac{3x-15}{|x-5|} = \lim_{x\to5^+}\frac{3x-15}{x-5} = \lim_{x\to5^+}\frac{3(x-5)}{x-5} = \lim_{x\to5^+} 3.$$

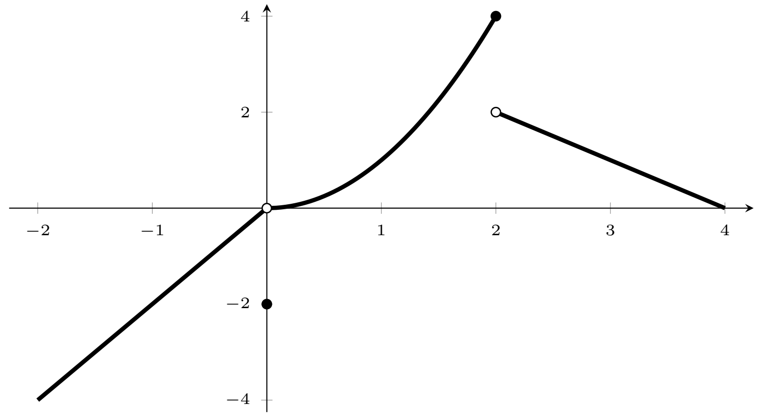
Since the limits from the left and right are not equal, the limit does not exist.

7. **a.**

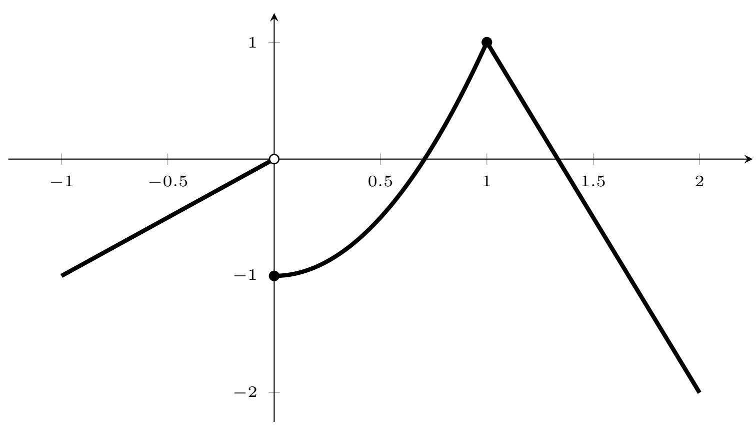
i. $\lim_{x\to 0^-} g(x) = 0$ **iii.** $g(0) = -2$ **v.** $\lim_{x\to 2^+} g(x) = 2$

ii. $\lim_{x\to 0} g(x) = 0$ **iv.** $\lim_{x\to 2^-} g(x) = 4$ **vi.** $\lim_{x\to 2} g(x)$ D.N.E.

b. The graph of the function is below.



8. f is discontinuous at $x = 0$ as $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$. Here it is continuous from the right. The graph of the function is below.



9. Since $x = 2$ is the only potential point of discontinuity, we want to examine continuity here. In fact, since f is continuous from the right at $x = 2$, we only need to find c so that $\lim_{x\to 2^-} f(x) = f(2)$

$$\begin{aligned}
\lim_{x\to 2^-} f(x) &= f(2) \\
\lim_{x\to 2^-} 3x^2 + cx &= c(2)^3 - 2(3) \\
3(2)^2 + c(2) &= 8c - 6 \\
12 + 2c &= 8c - 6 \\
\Rightarrow\ c &= 3.
\end{aligned}$$