§4.1 Maximum and Minimum Values

1. For $x > 0$, find the x-coordinate of the absolute minimum value of the function $f(x) = 8x \ln x - 6x$.

2. The function $g(x) = (2x + 5)e^{-6x}$ has one critical point. Find it.

3. Consider the function $h(t) = 8t^3 + 81t^2 - 42t - 8$ on $[-4, 2]$. Use the Extreme Value Theorem to find the absolute maximum and absolute minimum and the location of each.

§4.2 The Mean Value Theorem

4. Consider the function $f(x) = 4x^3 - 8x^2 + 7x - 2$ on the interval $[2, 5]$. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem to four decimal places.

5. Consider the function $g(z) = -z^3 - z^2 + 2z$ on the interval $[-2, 1]$. Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem to four decimal places.

§4.3 Derivatives and the Shapes of Graphs

6. For the function $f(x) = (2x + 5)e^{-6x}$, list the x-value(s) of the inflection point(s).

7. Suppose that $g(t) = 6t^5 - 4t^3$. Use interval notation to indicate where $g(t)$ is concave up and concave down. Justify your answer with the concavity test.

8. Suppose that $f(z) = \dfrac{e^z}{7 + e^z}$. Use interval notation to indicate where $f(z)$ is concave up and concave down. Justify your answer with the concavity test.

9. Suppose that $f(x) = 18x - 3\ln(2x), \quad x > 0$. Use interval notation to state where the function is concave up and concave down. Justify your answer with the concavity test.

§4.4 Curve Sketching

10. Let $f(x) = \dfrac{x - 1}{x^2}$.

a. State the domain of f.

b. Find the y- and x-intercepts of f.

c. Find any horizontal asymptotes of f.

d. Find any vertical asymptotes of f.

e. Find intervals of increase or decrease.

f. Find local maximum and minimum values.

g. Find intervals of concavity and inflection points.

h. Sketch the graph $y = f(x)$.

§4.5 Optimization Problems

11. A fence is to be built to enclose a rectangular area of 360 square feet. The fence along three sides is to be made of material that costs 7 dollars per foot and the fourth side costs 13 dollars per foot. Find the width (where width $W \leq$ length L) in feet of the enclosure that is most economical to construct. Round your answer to four decimal places.

12. A box is to be made out of a 15 cm by 20 cm piece of cardboard. Squares of side length x cm will be cut out of each corner, and then the ends and sides will be folded up to form a box with an open top. Find the height of the box that maximizes volume. Round your answer to two decimal places.

13. A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 13 - x^2$. What are the dimensions of such a rectangle with the greatest possible area? Round your answer to two decimal places.

§4.7 Antiderivatives

14. Given $f'(x) = 12\sin x - 6\cos x$ and $f(0) = 4$, find $f(x)$.

15. Find the particular antiderivative satisfying the following conditions: $f''(x) = e^x$; $f'(0) = 7$; $f(0) = -2$.

16. Find the general antiderivative for $f(x) = \dfrac{16}{1+x^2}$.

§5.1 Areas and Distance

17. Estimate the area under the graph of $3x^3 + 9$ from $x = -1$ to $x = 5$ by using 6 rectangles by finding a left hand approximation.

18. Estimate the area under the graph of $f(x) = 36x^2$ from $x = 0$ to $x = 6$ using 6 approximating rectangles and

a. right endpoints.

b. left endpoints.

In each part, show the sum you used.

§5.2 The Definite Integral

19. Evaluate the following integral by interpreting it in terms of areas: $\displaystyle\int_{-15}^{15} \sqrt{225 - x^2}\, dx$

20. Find a and b if $\displaystyle\int_a^b f(x)\, dx = \int_{14}^{37} f(x)\, dx - \int_{14}^{22} f(x)\, dx$.

21. Evaluate the following integral by interpreting it in terms of areas: $\displaystyle\int_{-7}^{7} (2 - |x|)\, dx$

1. $x = e^{-1/4}$

2. $x = -\frac{7}{3}$

3. The absolute minimum value is $-\frac{213}{16}$ and occurs at $t = \frac{1}{4}$. The absolute maximum value is 944 and occurs at $t = -4$.

4. $c = \frac{2}{3} + \frac{\sqrt{79}}{3} \approx 3.6294$

5. $c = -\frac{1}{3} + \frac{\sqrt{7}}{3} \approx 0.5486$ and
$c = -\frac{1}{3} - \frac{\sqrt{7}}{3} \approx -1.2153$

6. $x = -\frac{13}{6}$

7. Concave Down: $\left(-\infty, -\frac{1}{\sqrt{5}}\right)$, $\left(0, \frac{1}{\sqrt{5}}\right)$
Concave Up: $\left(-\frac{1}{\sqrt{5}}, 0\right)$, $\left(\frac{1}{\sqrt{5}}, \infty\right)$

8. Concave Down: $(\ln 7, \infty)$
Concave Up: $(-\infty, \ln 7)$

9. Concave Down: N/A
Concave Up: $(0, \infty)$

10. Let $f(x) = \dfrac{x-1}{x^2}$.

a. $(-\infty, 0) \cup (0, \infty)$

b. y-intercept: none
x-intercept: $x = 1$.

c. $y = 0$.

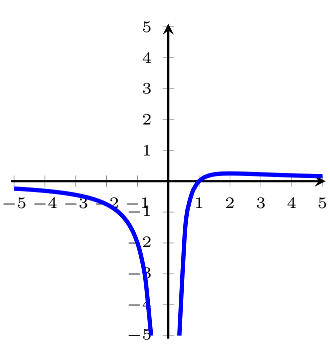
d. $x = 0$.

e. Increasing: $(0, 2)$
Decreasing: $(-\infty, 0)$, $(2, \infty)$

f. Local minimum: none
Local maximum: $\left(2, \frac{1}{4}\right)$

g. Inflection point: $x = 3$
Concave down: $(-\infty, 0)$, $(3, \infty)$
Concave up: $(0, 3)$

h.



11. $6\sqrt{7} \approx 22.6779\,\text{ft}$

12. $2.83\,\text{cm}$

13. $2\sqrt{\frac{13}{3}} \times \frac{26}{3}$ or 4.1633×8.666

14. $f(x) = -12\cos x - 6\sin x + 16$

15. $f(x) = e^x + 6x - 3$

16. $F(x) = 16\arctan(x) + C$

17. $L_6 = 351$

18. **a.** $R_6 = \sum_{i=1}^{6} 36(i)^2 = 3276$
b. $L_6 = \sum_{i=1}^{6} 36(i-1)^2 = 1980$

19. $a = \frac{225}{2}\pi$

20. $a = 22$, $b = 37$

21. -21