§1.3 The Limit of a Function

1. *Numerically or Algebraically* calculate the following limits exactly (if they exist):

a. $\lim\limits_{x \to 0} \cos\left(\frac{3\pi}{x}\right)$ **b.** $\lim\limits_{x \to 4} \frac{x-4}{\sqrt{x}-2}$

2. Sketch a graph of the following function, and evaluate the limits in (a)-(f), if they exist.

$$f(x) = \begin{cases} -x^2 & \text{if } x < -1 \\ 2 & \text{if } x = -1 \\ x^3 - 1 & \text{if } -1 < x < 2 \\ 9 - x & \text{if } x \geq 2 \end{cases}$$

a. $\lim\limits_{x \to -1^-} f(x)$ **c.** $\lim\limits_{x \to -1} f(x)$ **e.** $\lim\limits_{x \to 2^+} f(x)$

b. $\lim\limits_{x \to -1^+} f(x)$ **d.** $\lim\limits_{x \to 2^-} f(x)$ **f.** $\lim\limits_{x \to 2} f(x)$

§1.4 Calculating Limits

3. *Algebraically* calculate the exact limits (if they exist):

a. $\lim\limits_{h \to 0} \frac{\frac{7}{a+h} - \frac{7}{a}}{h}$ **b.** $\lim\limits_{h \to 0} \frac{3(x+h)^3 - 3x^3}{h}$ **c.** $\lim\limits_{t \to 2} \frac{t-2}{4-t^2}$

4. Suppose $\lim\limits_{x \to a} f(x) = -8$, $\lim\limits_{x \to a} g(x) = 4$, and $\lim\limits_{x \to a} h(x) = 0$. Determine the following limits (if they exist).

a. $\lim\limits_{x \to a} f(x) + g(x)$ **c.** $\lim\limits_{x \to a} f(x)g(x)$ **e.** $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$

b. $\lim\limits_{x \to a} g(x) - h(x)$ **d.** $\lim\limits_{x \to a} \frac{f(x)}{h(x)}$ **f.** $\lim\limits_{x \to a} \sqrt{f(x)}$

§1.5 Continuity

5. True or False? If false, provide a counter-example. Suppose f has domain $(-\infty, \infty)$.

a. If $\lim\limits_{x \to a} f(x)$ exists, then f is continuous at a.

b. If f is continuous at a, then $\lim\limits_{x \to a} f(x)$ exists.

6. Redefine the function value $g(0)$ below to make it continuous at $x = 0$.

$$g(x) = \begin{cases} \frac{3}{x} + \frac{2x+15}{x(x-5)} & \text{if } x \neq 0, 5 \\ \frac{1}{2} & \text{if } x = 0 \\ 18 & \text{if } x = 5 \end{cases}$$

7. For what value of the constant c is the following function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^4 + 18 & \text{if } x <= 3 \\ x^2 + 7cx & \text{if } x > 3 \end{cases}$$

§1.6 Limits Involving Infinity

8. Calculate the following limits exactly:

a. $\lim_{t\to\infty} \sqrt{36t^2 - 18t + 6}$

b. $\lim_{x\to-\infty} \dfrac{\sqrt{3x^2 + 49}}{2x - 7}$

9. Find all vertical and horizontal asymptotes of the function $g(x) = \dfrac{x-3}{x^2-9}$.

§2.1 Derivatives and Rates of Change

10. State the limit definition of "The derivative of f at a number a" in two different ways.

11. The limit $\lim_{h\to0} \dfrac{\sqrt{144+h} - 12}{h}$ represents the derivative of a function $f(x)$ at a number a. Determine both f and a.

12. The limit $\lim_{x\to2} \dfrac{x^4 + x - 18}{x - 2}$ represents the derivative of a function $f(x)$ at a number a. Determine both f and a.

13. The limit $\lim_{h\to0} \dfrac{(4+h)^3 - 64}{h}$ represents the derivative of a function $f(x)$ at a number a. Determine both f and a.

§2.2 The Derivative as a Function

14. State the limit definition of "The derivative of f."

15. True or False? If false, provide a counter-example.

a. If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

b. If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.

16. Using the *limit definition of the derivative*, find $\dfrac{df}{dx}$ where $f(x) = \dfrac{3}{x}$.

17. Consider the curve $y = 3 + x^3$. Using the *limit definition of the derivative*, find the equation of the tangent line to the curve at $x = 1$.

§2.3 Basic Differentiation Formulas

18. Let $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$. Find the derivative $\dfrac{df}{dx}$.

19. Let $h(s) = s^{4/5} - s^{2/3}$. Find the derivative $h'(s)$.

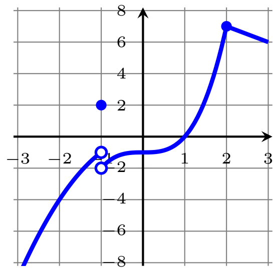
20. Find an equation of the normal line to the parabola $y = -x^2 + 5x + 3$ that is parallel to the line $2x + y = -4$.

21. Find the equation of the tangent line to the curve $y = \cos(x) + \sin(x)$ when $x = \frac{\pi}{4}$.

1.

- **a.** Does Not Exist.
- **b.** 4

2.



- **a.** -1
- **b.** -2
- **c.** Does Not Exist.
- **d.** 7
- **e.** 7
- **f.** 7

3.

- **a.** $-\dfrac{7}{a^2}$
- **b.** $9x^2$
- **c.** $-\dfrac{1}{4}$

4.

- **a.** -4
- **b.** 4
- **c.** -32
- **d.** Does Not Exist.
- **e.** -2
- **f.** Does Not Exist.

5.

- **a.** False. $f(x) = \dfrac{x^2-9}{x+9}$ at $x = 9$.
- **b.** True.

6.

$$g(x) = \begin{cases} \frac{3}{x} + \frac{2x+15}{x(x-5)} & \text{if } x \neq 0, 5 \\ -1 & \text{if } x = 0 \\ 18 & \text{if } x = 5 \end{cases}$$

7. $c = -\dfrac{3}{20}$

8.

- **a.** ∞
- **b.** $-\dfrac{\sqrt{3}}{2}$

9. Vertical asymptotes: $x = -3$. Horizontal asymptotes: $y = 1$.

10.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

and

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

11. $f(x) = \sqrt{x}$ and $a = 12$

12. $f(x) = x^4 + x$ and $a = 2$

13. $f(x) = x^3$ and $a = 4$.

14.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

15.

- **a.** True.
- **b.** False. $f(x) = |x|$ at $x = 0$.

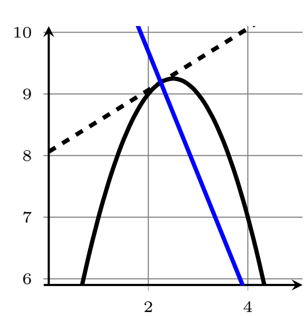
16. $-\dfrac{3}{x^2}$

17. $y = 3x + 1$

18. $f'(x) = 1 - 8x^{-3}$

19. $h'(s) = \dfrac{4}{5}s^{-1/5} - \dfrac{2}{3}s^{-1/3}$

20. $y = -2x + \dfrac{219}{16}$



21. $y = \sqrt{2}$