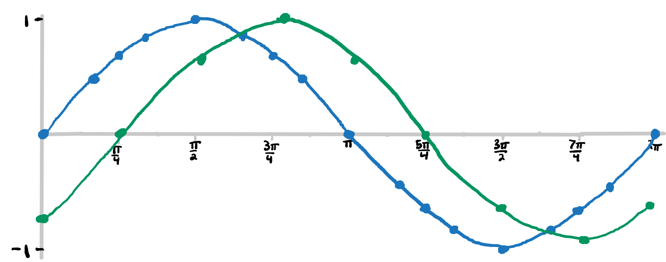
Section 4.5 (Continued)

Last time we saw how vertical and horizontal shrinks/stretches affected the graph of $y=\sin(x)$. What about horizontal shifts?

Ex $y=\sin\left(x-\frac{\pi}{4}\right)$ is a horizontal shift right by $\frac{\pi}{2}$, as expected

x	$\sin\left(x-\frac{\pi}{4}\right)$
0	$-\frac{\sqrt{2}}{2}$
$\frac{\pi}{6}$	-0.259
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	0.259
$\frac{\pi}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{2\pi}{3}$	0.966
$\frac{3\pi}{4}$	1
$\frac{5\pi}{6}$	0.966

x	$\sin\left(x-\frac{\pi}{4}\right)$
π	$\frac{\sqrt{2}}{2}$
$\frac{7\pi}{6}$	0.259
$\frac{5\pi}{4}$	0
$\frac{4\pi}{3}$	-0.259
$\frac{3\pi}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{3}$	-0.966
$\frac{7\pi}{4}$	-1
$\frac{11\pi}{6}$	-0.966



Graph of $y=\sin(x)$
Graph of $y=\sin\left(x-\frac{\pi}{4}\right)$

More generally, if $y=A\sin(Bx-C)=A\sin\left(B\left(x-\frac{C}{B}\right)\right)$, $A,B,C\neq 0$, the graph has amplitude A, period $\frac{2\pi}{B}$, and phase shift $\frac{C}{B}$.

Ex Determine the amplitude, period, and phase shift of $y=3\sin\left(2x-\frac{\pi}{2}\right)$. Then graph one period of the function.

Amplitude: $|3|=3$

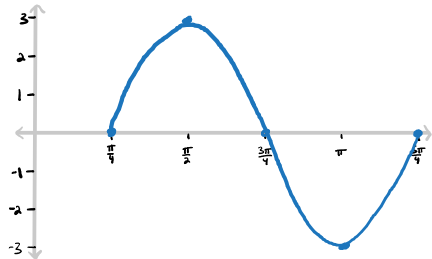
Period: $\frac{2\pi}{2}=\pi$

Phase shift: $\frac{\pi/2}{2}=\frac{\pi}{4}$

Recall that the x-intercepts lie at the end points of the period (so $x=\frac{\pi}{4}$, $x=\frac{\pi}{4}+\pi=\frac{5\pi}{4}$), and at the midpoint $\left(x=\frac{\pi}{4}+\frac{\pi}{2}=\frac{3\pi}{4}\right)$.

Since the graph is not reflected, the max is halfway between the first two intercepts (so $x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$) and the max is halfway between the second two (so $x=\frac{\pi}{4}+\frac{3\pi}{4}=\pi$).

Ex (Cont'd) So, the graph of one period of $y = 3\sin\left(2x - \frac{\pi}{2}\right)$ is

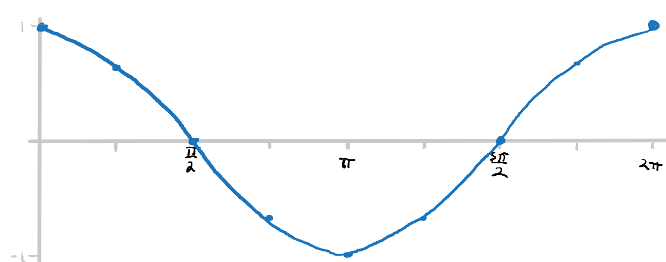


Procedure Recap:

1) Identify amplitude A, period $\frac{2\pi}{B}$, and phase shift $\frac{C}{B}$.
2) Identify x-intercepts, max, and min (these occur every $\frac{C}{B} + n\,\frac{\text{period}}{4}$, $n = 0, \dots, 4$).
3) For x-values above, plot the 5 points.
4) Connect w/ a smooth curve.

Graphing $y = \cos(x)$.

x	$\cos(x)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



We notice a few things: the x-intercepts, max, and min are all still at $\frac{\text{period}}{4}$ intervals, and the graph of $y = \cos(x)$ is the same as the graph of $y = \sin\left(x - \frac{\pi}{2}\right)$ (the latter is not surprising as sine and cosine are cofunctions).

NOTE: FOR $y = A\cos(Bx - C)$, $A, B, C \neq 0$, THE AMPLITUDE, PERIOD, AND PHASE SHIFT ARE ALL STILL DEFINED IN THE SAME WAY.

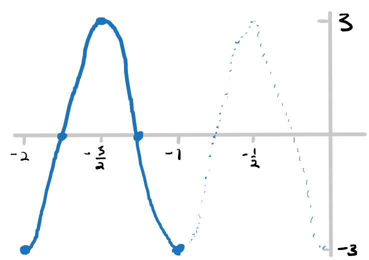
Ex $y = -3x(2\pi x + 4\pi)$ NOTE THAT THE NEGATIVE LEADING TERM IS A REFLECTION ABOUT THE x-AXIS.

AMPLITUDE: $|-3| = 3$

PERIOD: $\frac{2\pi}{2\pi} = 1$

PHASE SHIFT: $\frac{-4\pi}{2\pi} = -2$

x	-2	$-2+\frac{1}{4}$	$-2+\frac{2}{4}$	$-2+\frac{3}{4}$	$-2+\frac{4}{4}$
$-3\cos(2\pi x + 4\pi)$	-3	0	3	0	-3



THE PROCEDURE FOR GRAPHING BOTH $y = A\sin(Bx - C)$ AND $y = A\cos(Bx - C)$.

REMARK. ALL TRANSFORMATIONS OF $\sin(x)$ AND $\cos(x)$ BEHAVE AS YOU'D EXPECT, WE JUST DON'T HAVE ANY COOL NAMES FOR VERTICAL SHIFTS.

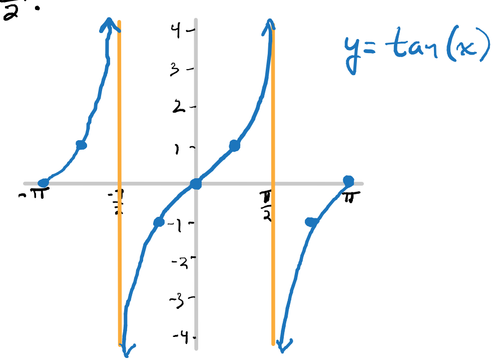
Ex $y = 3\sin\left(\frac{2\pi}{365}(x - 79)\right) + 12$ REPRESENTS THE NUMBER OF HOURS OF DAYLIGHT IN BOSTON, x-DAYS AFTER JANUARY 1.

THE AMPLITUDE IS $|3| = 3$, THE PERIOD IS $\frac{2\pi}{(2\pi/365)}$, AND THE PHASE SHIFT IS 79. THE LONGEST DAY OF THE YEAR OCCURS AT $x = 79 + \frac{365}{4}$ days, WHICH IS JUNE 20th. THE AMOUNT OF DAYLIGHT IS 15 HOURS. THE SHORTEST DAY OF THE YEAR IS AT $x = 79 + \frac{3(365)}{4}$ days, WHICH IS DECEMBER 19. THE AMOUNT OF DAYLIGHT IS ONLY 9 HOURS.

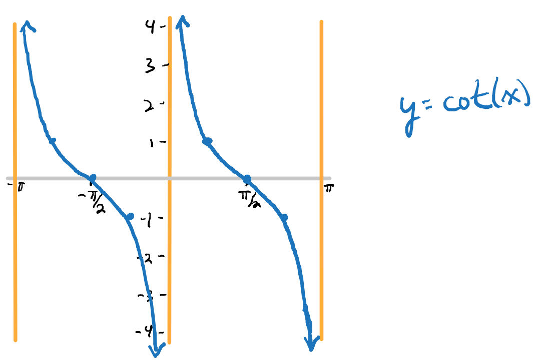
SECTION 4.6

THIS WILL NOT BE ON THE EXAM, BUT I THINK IT'S GOOD TO SEE AT SOME POINT.

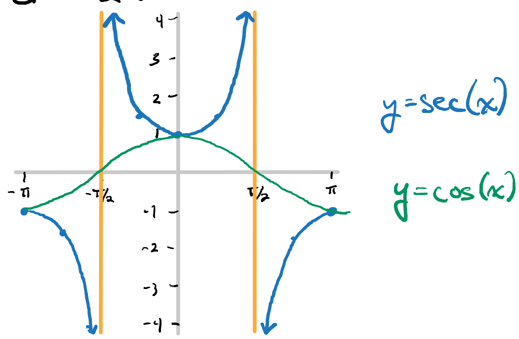
SINCE $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$, THE DOMAIN IS
$\{\theta \mid \cos(\theta) \neq 0\} = \{\theta \mid \theta \neq \frac{2k+1}{2}\pi$, k AN INTEGER$\}$. INDEED, TANGENT ALSO HAS VERTICAL ASYMPTOTES AT EACH ODD MULTIPLE OF $\frac{\pi}{2}$.



SINCE $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$, THE DOMAIN IS
$\{\theta \mid \sin(\theta) \neq 0\} = \{\theta \mid \theta \neq k\pi$, k AN INTEGER$\}$. INDEED, $y = \cot(x)$ HAS VERTICAL ASYMPTOTES AT EACH MULTIPLE OF π.



SINCE $\sec(\theta) = \frac{1}{\cos(\theta)}$, $y = \sec(x)$ HAS THE SAME DOMAIN AS $y = \tan(x)$. INDEED, IT HAS VERTICAL ASYMPTOTES AT EACH ODD MULTIPLE OF $\frac{\pi}{2}$.



SINCE $\csc(\theta) = \frac{1}{\sin(\theta)}$, $y = \csc(x)$ HAS THE SAME DOMAIN AS $y = \cot(x)$. INDEED, $y = \csc(x)$ HAS VERTICAL ASYMPTOTES AT MULTIPLES OF π.