Section 4.2 (Continued)

Def A function is called periodic if there exists a real number $p > 0$ s.t. $f(x+p) = f(x)$ for all x in the domain of f. The number p is called the period of f.

Periodic Properties of Sine, Cosine, Cosecant, and Secant

$\sin(\theta + 2\pi) = \sin(\theta)$ and $\cos(\theta + 2\pi) = \cos(\theta)$

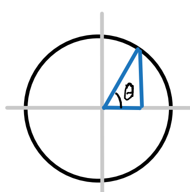
$\csc(\theta + 2\pi) = \csc(\theta)$ and $\sec(\theta + 2\pi) = \sec(\theta)$

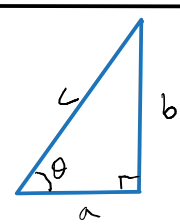
Each of these has period 2π.

Periodic Properties of Tangent and Cotangent

$\tan(\theta + \pi) = \tan(\theta)$ and $\cot(\theta + \pi) = \cot(\theta)$

Section 4.3





By merely drawing a triangle, we get

Def Each trig function can be defined in terms of the right triangle above:

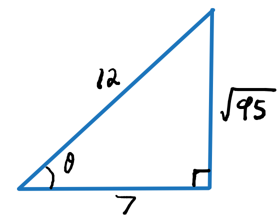
$\sin(\theta) = \frac{b}{c}$ $\qquad$ $\csc(\theta) = \frac{c}{b}$

$\cos(\theta) = \frac{a}{c}$ $\qquad$ $\sec(\theta) = \frac{c}{a}$

$\tan(\theta) = \frac{b}{a}$ $\qquad$ $\cot(\theta) = \frac{a}{b}$

Notice that, when $c = 1$, these exactly align with our unit circle definitions.

Knowing only one trig function value, we can determine the others.

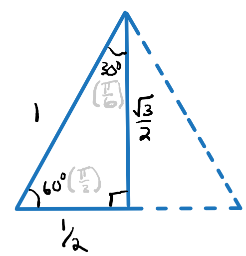


Suppose $\sec(\theta) = \frac{12}{7}$. Then $7^2 + b^2 = 12^2$ $\Rightarrow b^2 = 144 - 49$ $\Rightarrow b = \sqrt{95}$ is the length of our missing side. So

$\sin(\theta) = \frac{\sqrt{95}}{12}$ $\qquad \csc(\theta) = \frac{12}{\sqrt{95}}$

$\cos(\theta) = \frac{7}{12}$ $\qquad \cot(\theta) = \frac{7}{\sqrt{95}}$

$\tan(\theta) = \frac{\sqrt{95}}{7}$



An important angle is $60^\circ = \frac{\pi}{3}$. If you recall, a triangles internal angles sum to $180^\circ = \pi$, and in an equilateral triangle, each angle is $\frac{\pi}{3}$. So, consider the equilateral triangle w/ side length 1, and examine the right triangle formed by cutting it in half. With the Pythagorean theorem, the remaining side length is $\frac{\sqrt{3}}{2}$. Thus

$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ $\qquad \csc(\frac{\pi}{3}) = \frac{2\sqrt{3}}{3}$ $\qquad \sin(\frac{\pi}{6}) = \frac{1}{2}$ $\qquad \csc(\frac{\pi}{6}) = 2$

$\cos(\frac{\pi}{3}) = \frac{1}{2}$ $\qquad \sec(\frac{\pi}{3}) = 2$ $\qquad \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ $\qquad \sec(\frac{\pi}{6}) = \frac{2\sqrt{3}}{3}$

$\tan(\frac{\pi}{3}) = \sqrt{3}$ $\qquad \cot(\frac{\pi}{3}) = \frac{\sqrt{3}}{3}$ $\qquad \tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$ $\qquad \cot(\frac{\pi}{6}) = \sqrt{3}$

Def Two angles are **complements** if they sum to 90°. Two functions are called **cofunctions** if $f(\theta) = g(90^\circ - \theta)$.

Ex $30^\circ = \frac{\pi}{6}$ and $60^\circ = \frac{\pi}{3}$ are complements.

sin and cos are cofunctions.

Remark: "cosine" is short for "complement's sine."

Cofunction Identities (if θ is in radians, replace $90°$ w/ $\frac{\pi}{2}$)

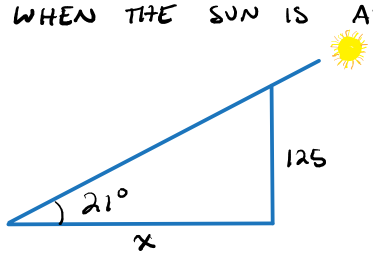
$\sin(\theta) = \cos(90° - \theta)$ $\qquad$ $\csc(\theta) = \sec(90° - \theta)$

$\cos(\theta) = \sin(90° - \theta)$ $\qquad$ $\sec(\theta) = \csc(90° - \theta)$

$\tan(\theta) = \cot(90° - \theta)$ $\qquad$ $\cot(\theta) = \tan(90° - \theta)$

Ex Since sine and cosine are cofunctions, $\cos(38°)$ and $\sin(52°)$ have the same value. Also, $\sec\left(\frac{\pi}{12}\right) = \csc\left(\frac{5\pi}{12}\right)$.

Ex Find the length of the shadow cast by a 125 ft tower when the sun is at $21°$ above the horizon.



$$\tan\left(\frac{\pi}{8}\right) = \frac{125 \text{ ft}}{x}$$

$$\Rightarrow \quad x = \frac{125 \text{ ft}}{\tan(21°)} \approx 325.64 \text{ ft}.$$