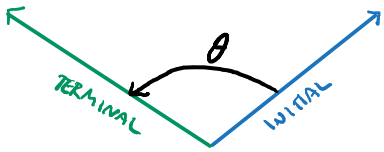
Section 4.1

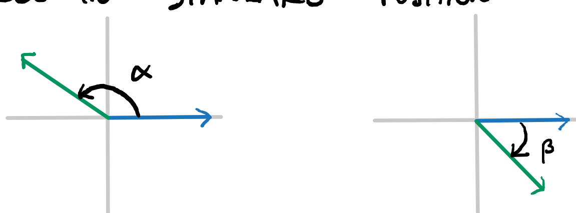
Def An **angle** is formed by two rays that share a common endpoint. One ray is called the **initial side** and the other is called the **terminal side**. The common endpoint is called the **vertex**.



An angle's direction and amount of rotation are from the initial side to the terminal side. Angles are usually labeled w/ Greek letters.

Def An angle is in **standard position** if the initial side is along the positive x-axis and the vertex is at the origin.

Ex Angles in standard position

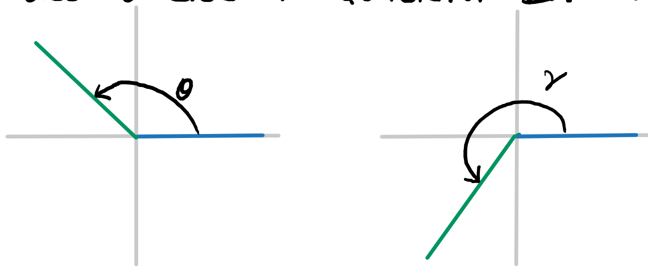


Def An angle is **positive** if the rotation is CCW, and **negative** if the rotation is CW.

Ex In the previous example, α is positive, β is negative.

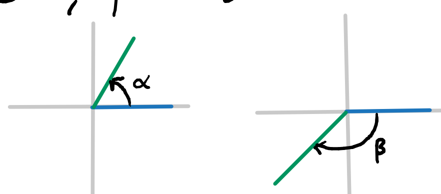
Def We say that an angle **lies in the quadrant** where its terminal side lies.

Ex Angle θ lies in Quadrant II. γ lies in Quadrant III



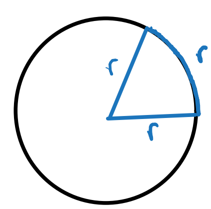
Def One **degree** (written 1°), is a measurement of the amount of rotation of an angle, and is $1/360^{th}$ of a complete rotation.

Ex $\alpha = 60^\circ$, $\beta = -135^\circ$



Def One **radian** (written just 1 or 1 rad) is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

In simpler terms, it's the measure of the angle that makes the three sides (two straight, one curved) in the picture to the right have the same length.



We can lay $2\pi \approx 6.28$ of these arcs along the circle, so this means that, in radians, a full rotation has measure 2π.

So, 2π rad $= 360°$ $\Rightarrow$ π rad $= 180°$. Dividing one side by the other leads us to the following:

Converting Between Radians and Degrees:

- Radians to Degrees: multiply the angle by $\frac{180°}{\pi}$.
- Degrees to Radians: Multiply the angle by $\frac{\pi}{180°}$.

Ex

$$45° = 45°\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}\text{ rad}$$

$$198° = 198°\left(\frac{\pi}{180°}\right) = \frac{11\pi}{10}$$

$$-\frac{\pi}{6} = -\frac{\pi}{6}\left(\frac{180°}{\pi}\right) = -30°$$

$$1.71\pi = 1.71\pi\left(\frac{180°}{\pi}\right) = 307.8°$$

Def Two angles are **coterminal** if they differ by a multiple of $360°$ or 2π rad.

Ex $180°$ and $-180°$ are coterminal: $180° = -180° + 360°$.

$\frac{25\pi}{6}$ and $\frac{\pi}{6}$ are coterminal: $\frac{25\pi}{6} = \frac{\pi}{6} + 2(2\pi) = \frac{\pi}{6} + \frac{24\pi}{6}$

Section 4.2

Def The **unit circle** is the circle of radius 1, centered at the origin.

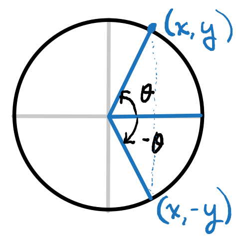
Def If θ is an angle and $P(x,y)$ is the corresponding point on the unit circle, we have the following functions:

$\sin(\theta) = y$; $\cos(\theta) = x$; $\tan(\theta) = \frac{y}{x}$, $x \neq 0$;

$\csc(\theta) = \frac{1}{y}$, $y \neq 0$; $\sec(\theta) = \frac{1}{x}$, $x \neq 0$; $\cot(\theta) = \frac{x}{y}$, $y \neq 0$.

Since θ can be any real number, the domain for sine and cosine is $(-\infty, \infty)$. Since $x = \cos\theta$ and $y = \sin\theta$, the range for cosine and sine is $[-1, 1]$.

Notice that on the unit circle, when we plug in θ and $-\theta$, we have that the x-values stay the same, but the y-values change sign. So in terms of our trig functions:



$$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$
$$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$$
$$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\tan(\theta)$$

This means that cos and sec are <u>even</u>; and sin, csc, tan, cot are <u>odd</u>.

Relationships between trig functions:

Each of these follows from the definitions.

$$\sin(\theta) = \frac{1}{\csc(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$
$$\cos(\theta) = \frac{1}{\sec(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{1}{\cot(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\tan(\theta)}$$

This means that, given some value for any two trig functions, we can determine the values for the other four.