1. Let x be the number of years since 2000 and y be the population (in millions). Then we have the points $(1, 48.68)$ and $(13, 45.49)$ on the graph of the function, so the slope of the line is

$$m = \frac{45.49 - 48.68}{13 - 1} = \frac{-3.19}{12} = -\frac{319}{1200}$$

and the equation of the line in *point-slope form* is given by

$$(y - 45.49) = -\frac{319}{1200}(x - 1).$$

2.

a. $f(g(x)) = -\left(\frac{1}{x+2}\right)^2 - 2\left(\frac{1}{x+2}\right) + 1.$
The domain of $f \circ g$ is $(-\infty, -2) \cup (2, \infty)$ or $\{x \mid x \neq 2\}$

b. $g(f(x)) = \frac{1}{(-x^2 - 2x + 1) + 2} = \frac{1}{-x^2 - 2x + 3} = \frac{1}{(1-x)(x+3)}.$
The domain of $g \circ f$ is $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$, or $\{x \mid x \neq -3, 1\}$.

3.

a. By the Rational Root Theorem, possible roots are $\pm 1, \pm 2, \pm 3, \pm 6$.

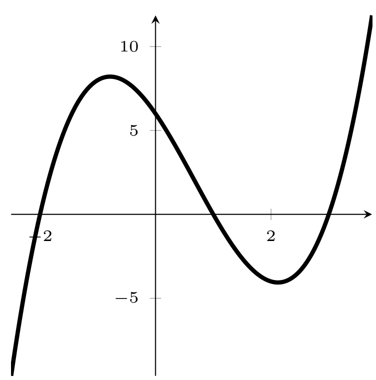
b. By plugging in the possible roots from part (a), we see that the roots are $-2, 1, 3$.

c. Since $r(x) = x^3 - 2x - 5x + 6 = (x+2)(x-1)(x-3)$, each root has multiplicity 1

d. By the leading coefficient test, since 3 is odd and $1 > 0$, the graph falls to the left and rises to the right.

e. The y-intercept is $r(0) = 6$.

f.



4. Let $g(x) = \frac{x^2 + 5x + 6}{(x+3)(x-2)}$.

a. The domain of g is $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$, or $\{x \mid x \neq -3, 2\}$.

b. y-intercept: $g(0) = \frac{6}{-6} = -1$.

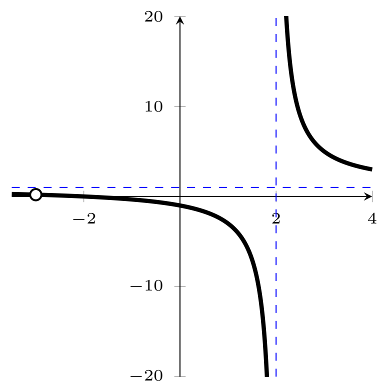
c. Roots occur when the numerator is 0. So $x^2 + 5x + 6 = (x+3)(x+2)$, so they can only occur at $x = -3, -2$. But -3 is not in the domain, so $g(x)$ has only one root at $x = -2$.

d. Since $x = -3$ is the only value that makes both the numerator and denominator zero, there is a hole at $x = -3$. Since $g(x)$ looks like $\frac{x+2}{x-2}$ (except with a different domain) the hole on the graph is at the point $(-3, \frac{1}{5})$.

e. Since $x = 2$ does not correspond to a hole, it must be the vertical asymptote.

f. Since the numerator and denominator have the same degree, the horizontal asymptote occurs at $y = \frac{1}{1} = 1$.

g.



5. Suppose the cost for Yamaha to manufacture xylophones is modeled by the function $c(x) = x^2 - 2x + 100$ where x is the number of xylophones manufactured.

a. By completing the square

$$\begin{aligned} c(x) &= x^2 - 2x + 100 \\ &= \left(x^2 - 2x\right) + 100 \\ &= \left(x^2 - 2x + (-1)^2 - (-1)^2\right) + 100 \\ &= \left(x^2 - 2x + (-1)^2\right) - (-1)^2 + 100 \\ &= (x-1)^2 + 99. \end{aligned}$$

In standard form, we see that the minimum cost (which corresponds to the vertex) occurs at $x = 1$, so Yamaha produces should only produce 1 xylophone to minimize cost.

b. The minimum cost is \$99.

c. The average rate of change in cost is

$$\frac{c(9) - c(5)}{9 - 5} = \frac{163 - 115}{9 - 5} = \frac{48}{4} = 12 \text{dollars per xylophone.}$$

6. Let $f(x) = \dfrac{1}{x^2 + 4}$

a. Since the denominator is never zero, the domain is $(-\infty, \infty)$ or $\mathbb{R}$. The range of $x^2 + 4$ is $[4, \infty)$, so the range of $\frac{1}{x^2+4}$ is $(0, \frac{1}{4}]$.

b. f passes the vertical line test, so it is a function.

c. To find the inverse,

$$\begin{aligned} x &= \frac{1}{y^2+4} \\ \frac{1}{x} &= y^2+4 \\ \frac{1}{x} - 4 &= y^2 \\ \Rightarrow f^{-1}(x) = y &= \pm\sqrt{\frac{1}{x} - 4}. \end{aligned}$$

d. f^{-1} is not a function because it fails the vertical line test (that is, for any $x \neq \frac{1}{4}$, $f^{-1}(x)$ has two outputs).

7. Let $g(x) = 6x^2 + 5x - 17$.

a. For $h \neq 0$, difference quotient is

$$\begin{aligned} \frac{g(x+h) - g(x)}{h} &= \frac{(6(x+h)^2 + 5(x+h) - 17) - (6x^2 + 5x - 17)}{h} \\ &= \frac{6(x^2 + 2xh + h^2) + 5(x+h) - 17 - 6x^2 - 5x + 17}{h} \\ &= \frac{6x^2 + 12xh + 6h^2 + 5x + 5h - 17 - 6x^2 - 5x + 17}{h} \\ &= \frac{12xh + 6h^2 + 5h}{h} \\ &= 12x + 6h + 5. \end{aligned}$$

b. Since $g(1) = 6(1) + 5(1) - 17 = -6$ and $g(2) = 6(4) + 5(2) - 17 = 17$, by the Intermediate Value Theorem, there exists a root of g between $x = 1$ and $x = 2$.

8. Suppose $w(x) = (x+1)^3 - 4$.

a. First option: $f(x) = x - 4$ and $g(x) = (x+1)^3$.
Second option: $f(x) = x^3 - 4$ and $g(x) = x + 1$.

b. Transformations: horizontal shift left 1 and vertical shift down 4, in any order.

9. Algebraically simplify and rewrite each of the following complex numbers in standard form $a + bi$:

a.

$$\begin{aligned} (2 - i) + (4 + 7i) &= (2 + 4) + (-1 + 7)i \\ &= 6 + 6i \end{aligned}$$

b.

$$\begin{aligned} (4 + 2i) - (3i) &= (4 - 0) + (2 - 3)i \\ &= 4 - i \end{aligned}$$

c.

$$\begin{aligned}(2+9i)(3+2i) &= 6+4i+27i+18i^2\\ &= 6+4i+27i-18\\ &= -12+31i\end{aligned}$$

d.

$$\begin{aligned}\frac{6+8i}{2-7i} &= \frac{6+8i}{2-7i}\left(\frac{2+7i}{2+7i}\right)\\ &= \frac{(6+8i)(2+7i)}{2^2+7^2}\\ &= \frac{12+42i+16i+56i^2}{53}\\ &= \frac{12+42i+16i-56}{53}\\ &= \frac{-44+58i}{53}\\ &= -\frac{44}{53}+\frac{58i}{53}\end{aligned}$$