A Bit About Difference Quotients

Recall that, the slope of a line between two points (x_1, y_1) and (x_2, y_2) is the number m given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

To see how this ties into the difference quotient, let f be some arbitrary function. Then for some constant number $h \neq 0$, consider the points $(x, f(x))$ and $(x+h, f(x+h))$ on the graph of the function. By substituting into the above equation, the slope of the line between these two points is given by

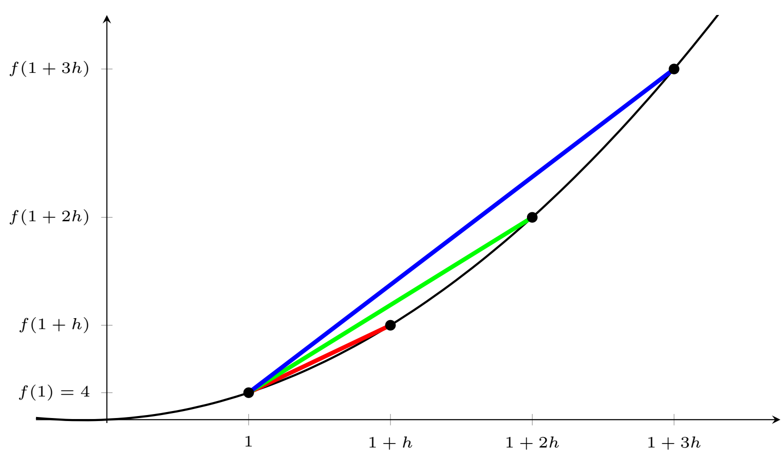
$$m = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}.$$

The expression on the right is the difference quotient, and it is just the slope of the line between these two points! So the question is, what does h have to do with anything? To motivate this, it might be useful to look at the particular example we had in class. We had the function $f(x) = 3x^2 + x$, and saw that the difference quotient was $3h + 6x + 1$.

Now let's fix the x-value for this difference quotient, say $x = 1$. Then $f(1) = 4$ and our difference quotient becomes

$$\frac{f(1+h) - f(1)}{h} = 3h + 7.$$

This expression on the right is a function of the variable h, and because it was derived from a difference quotient of f, this new function of h must be related to f somehow. Indeed, this function describes how the slope of the line from $(1, 4)$ to the point $(1+h, f(1+h))$ changes as we change h. Notice that, as h gets bigger, the slope of the line gets bigger as well. Visually, here's what's happening:



We'll see these difference quotients again soon when discussing average rates of change.